Circular Shift of a sequence

Let us consider length-N sequences defined for $0 \le n \le N-1$. Such sequences have sample values equal to zero for $n < 0$ and $n \ge N$.

For an arbitrary integer n_0, the shifted sequence $x_1[n] = x[n-n_0]$, may no longer be defined over the range $0 \le n \le N-1$.

This brings the requirement for an other type of shift that will keep the shifted sequence always in the range $0 \le n \le N-1$.

We define a new shift type known as the "*circular shift*".

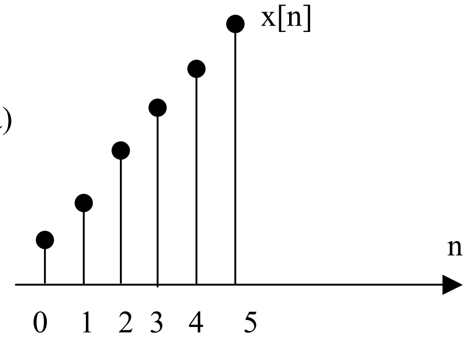
$$x_c[n] = x[\langle n-n_0 \rangle_N]$$

$$\text{where,} \quad \langle m \rangle_N = m \text{ modulo } N$$

for $n_0 > 0$ (right circular shift) the equations below apply:

$$x_c[n] = \begin{cases} x[n-n_0] & n_0 \le n \le N-1 \\ x[N-n_0+n] & 0 \le n \le n_0 \end{cases}$$

(a)

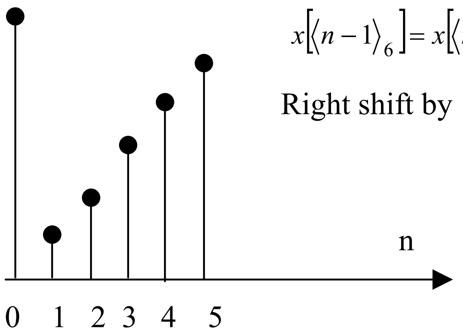


left shift by 5 units

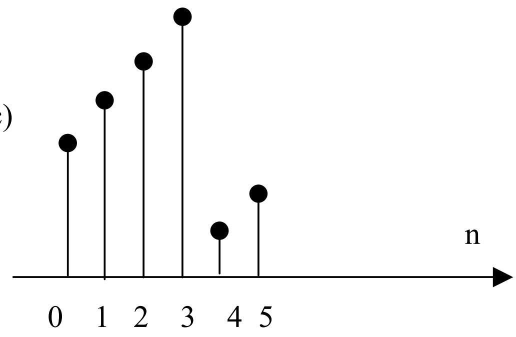
$$x[\langle n-1 \rangle_6] = x[\langle n+5 \rangle_6]$$

Right shift by 1 unit

(b)



(c)



$$x[\langle n-4\rangle_6] = x[\langle n+2\rangle_6]$$

It can be seen from (b) and (c) that right circular shift by n_0 is equivalent to a left circular shift by $(N - n_0)$.

More over a circular shift n_0 greater than N is equivalent to circular shift by $\langle n_0 \rangle_N$

Circular Convolution

Circular convolution between two length N sequences can be carried out as shown by the expression below:

$$y_C[n] = \sum_{m=0}^{N-1} g[m]h[\langle n-m\rangle_N]$$

Since the above operation involves two length-N sequences it is referred to as the N-point circular convolution and denoted by:

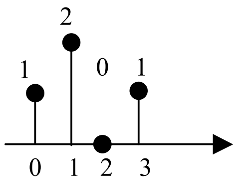
$$y_C[n] = g[n] \textcircled{N} h[n]$$

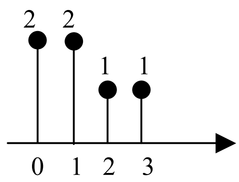
As in linear convolution circular convolution is commutative.

i.e. $$g[n] \textcircled{N} h[n] \equiv h[n] \textcircled{N} g[n]$$

Example:
Determine the 4-point circular convolution of the two length-4 sequences g[n] and h[n] given by:

$$g[n]=\{1,2,0,1\} \qquad \textit{and} \qquad h[n]=\{2,2,1,1\}$$





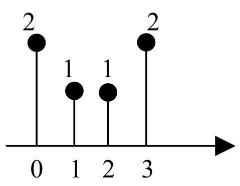
From the definition of the circular convolution:

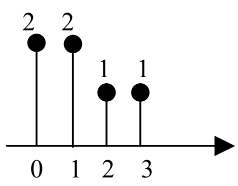
$$y_c[n]=g[n]\,④\,h[n]=\sum_{m=0}^{3} g[m]h\left[\langle n-m\rangle_N\right] \qquad 0\le n\le 3$$

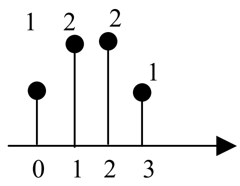
Therefore:

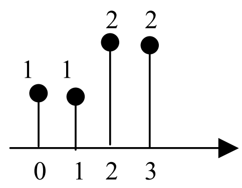
$$y_c[0]=\sum_{m=0}^{3} g[m]h\left[\langle -m\rangle_N\right] \qquad 0\le n\le 3$$

The circular time-reversed sequence $h\left[\langle -m\rangle_4\right]$ is as shown below:









$n=0$ $h\left[\langle -m\rangle_4\right]$ $\qquad$ $n=1$ $h\left[\langle 1-m\rangle_4\right]$ $\qquad$ $n=2$ $h\left[\langle 2-m\rangle_4\right]$ $\qquad$ $n=3$ $h\left[\langle 3-m\rangle_4\right]$

By performing the product of g[m] with $h\left[\langle -m\rangle_4\right]$ for each value of m in the range $0\le m\le 3$
And summing the products we get:

$$\begin{aligned} y_C[0] &= g[0]\cdot h[0]+g[1]\cdot h[3]+g[2]\cdot h[2]+g[3]\cdot h[1] \\ &= (1\times 2)+(2\times 1)+(0\times 1)+(1\times 2)=6 \end{aligned}$$

$$y_C[1] = \sum g[m]h[\langle 1-m \rangle_4]$$

$$y_C[1] = g[0]h[1] + g[1]\cdot h[0] + g[2]\cdot h3 + g[3]\cdot h[2]$$
$$= (1\times 2) + (2\times 2) + (0\times 1) + (1\times 1) = 7$$

$$y_C[2] = g[0]h[2] + g[1]\cdot h[1] + g[2]\cdot h[0] + g[3]\cdot h[3]$$
$$= (1\times 1) + (2\times 2) + (0\times 2) + (1\times 1) = 6$$

$$y_C[3] = g[0]h[3] + g[1]\cdot h[2] + g[2]\cdot h[1] + g[3]\cdot h[0]$$
$$= (1\times 1) + (2\times 1) + (0\times 2) + (1\times 2) = 5$$

Ans: $y_C[N] = \{6,7,6,5\}$