<table>
<thead>
<tr>
<th>Quantized Sample Voltage</th>
<th>Gray Code Word</th>
<th>Natural Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>+7</td>
<td>1 1 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>+5</td>
<td>1 1 1</td>
<td>0 0 1</td>
</tr>
<tr>
<td>+3</td>
<td>1 0 1</td>
<td>0 1 0</td>
</tr>
<tr>
<td>+1</td>
<td>1 0 0</td>
<td>0 1 1</td>
</tr>
<tr>
<td>-1</td>
<td>0 0 0</td>
<td>1 0 0</td>
</tr>
<tr>
<td>-3</td>
<td>0 0 1</td>
<td>1 0 1</td>
</tr>
<tr>
<td>-5</td>
<td>0 1 1</td>
<td>1 1 0</td>
</tr>
<tr>
<td>-7</td>
<td>0 1 0</td>
<td>1 1 1</td>
</tr>
</tbody>
</table>

*Note:* There is a one bit change per transition in the Gray-Code representation. Also the wrap-around from code word-7 to code-word 0. The wrap-around makes Gray encoder cyclic in nature.

**Practical PCM Circuits**

Three popular techniques are used to implement the analog-to-digital converter (ADC) encoding operation.

1. Counting or rump (INTERSIL ICL 7/26 CMOS)
2. Serial or successive approximation (NS ADC0804 8-bit ADC)
3. Parallel or flash encoders. (RCA CA3318 b-bit ADC)

All chips listed above have parallel digital outputs that correspond to the digital word that represents the analog sample value.

For generation of PCM we need to convert this to a serial form for transmission. Devices that achieve this are known as serial-input-output (SIO) chips.

Three types of SIO chips are available:

1) Universal Asynchronous Receiver/Transmitter (UART)
2) Universal synchronous Receiver/Transmitter (USRT)
3) Universal Synchronous/Asynchronous Receiver/Transmitter (USART)

**Bandwidth of PCM**

The spectrum of PAM signals was obtained in terms of the spectrum of the input analog signal.

This is not the case with PCM.

The bandwidth of PCM signals depend on the bit-rate and the pulse-shape.

From Fig. 3-8 bit rate is:

\[ R = n f_s \]

\( n = \) number of bits in the PCM word \((M = 2^n)\)
\( f_s = \) sampling rate

The dimensionality theorem says that bandwidth of the PCM waveform is bounded by:

\[ B_{PCM} \geq \frac{1}{2} R = \frac{1}{2} n f_s \]

The bandwidth also depends on the pulse shape which is discussed in detail in section 3.5 of chapter. If a rectangular shape with polar NRZ line coding is used the first null bandwidth is:

\[ B_{PCM} = R = n f_s \]

If \( f_s = 2B \) \( \Rightarrow \) \( 3 \times 2B = 6B \)
\( n=3 \)

\( f_s = 2B \) \( \Rightarrow \) \( 5 \times 2B = 10B \)
\( n=5 \)

We note that as \( n \) or \( f_s \) increases PCM signals bandwidth will increase.
**Effects of Noise**
The analog signal that is recovered at the PCM system output is corrupted by noise. Two main effects cause this noise:

- **Quantization Noise**
  - Overload noise
  - Random noise
  - Granular noise
  - Hunting noise

- **Bit Errors in the Recovered PCM Signal**
  - Channel noise
  - Improper channel filtering

The signal-to-noise ratio of the recovered analog peak and average signal power to the total average power is given by

\[
\left( \frac{S}{N} \right)_{pk\_out} = \frac{3M^2}{1 + 4(M^2 - 1)P_e}
\]

\[
\left( \frac{S}{N} \right)_{out} = \frac{M^2}{1 + 4(M^2 - 1)P_e}
\]

$P_e$ is the probability of bit error in the recovered binary PCM signal at the receiver before it is converted back into an analog signal.

If we assume that there are no bit errors resulting from channel noise (i.e. $P_e = 0$) and no ISI, then the peak SNR resulting from only quantizing errors is:

\[
\left( \frac{S}{N} \right)_{peak\_out} = 3M^2
\]

\[
\left( \frac{S}{N} \right)_{out} = M^2
\]
Recalling that $M = 2^n$ we may express equations in decibels as

$$\left( \frac{S}{N} \right)_{dB} = 6.02n + \alpha$$

where, $n$ is the number of bits in the PCM word

$\alpha = 4.77$ for the peak SNR, and $\alpha = 0$ for the average SNR.

This equation is called the 6dB rule. An additional 6-dB improvement in SNR is obtained for each bit added to the PCM word.

Show Example 3-1: Design of a PCM System (pages 145-146)

Show Example (not in book)

**Digital Signaling, Line Codes & Spectra**

How do we estimate the bandwidth of a waveform?

The baud (symbol rate) is: $D = N / T_0$ symbols/s

The bit rate is: $R = n / T_0$ bits/s

- Bandwidth estimation

$$B \geq \frac{N}{2T_0} = \frac{1}{2} D \text{ Hz}$$
Binary & Multilevel Signalling
A waveform that represents a binary signal is described by the N-dimensional orthogonal series where the coefficients, $w_k$, take on binary values.

Show
Example 3-3: Binary signaling (page 153)

Multilevel Signaling:
For binary case the lower bound bandwidth was $B = \frac{N}{2T_0}$

For $N = 8$ pulses and $T_0 = 8ms$ $B = 500$ Hz.

This means that bandwidth can be reduced if $N$ is reduced.

$N$ can be reduced by letting $w_k$’s take on $L > 2$ possible values. This process is called multi-level signaling.

Show
Example 3-4: L=4 Multilevel signal (page 155)
**Binary Line Coding**

Binary 1s and 0s, such as in PCM signaling, may be represented in various serial-bit signaling-formats *called line codes*. Some popular line codes are:

1. Unipolar Signaling
2. Polar Signaling
3. Bipolar (Pseudoternary) Signaling
4. Manchester signaling

![Figure 3-15 Binary signaling formats.](image-url)
**Unipolar Signaling:** In positive logic unipolar signaling the binary 1 is represented by a high level (+A volts) and a binary 0 by a zero level. This type of signaling is also called *on-off keying*.

**Polar Signaling:** Binary 1’s and 0’s are represented by equal positive and negative levels.

**Bipolar (Pseudoternary) Signaling:** Binary 1’s are represented by alternately positive or negative values. The binary 0 is represented by a zero level. The term pseudoternary refers to the use of three encoded signal levels to represent two-level (binary) data.

**Manchester Signaling:** Each binary 1 is represented by a positive half-bit period pulse followed by a negative half-bit period pulse. Similarly a binary 0 is represented by a negative half-bit period pulse followed by a positive half-bit period pulse. This type of signaling is also called *split-phase encoding*. 
Figure 3.16 PSD for line codes (positive frequencies shown).
Regenerative Repeaters

When a line code digital signal (such as PCM) is transmitted over a hardwire channel (such as twisted-pair telephone cables), it is attenuated, filtered, and corrupted by noise. Consequently for long lines the data can not be recovered at the receiving end unless repeaters are placed in cascade along the line and at the receiver as shown below:
The repeaters amplify and clean up the signal periodically.

A simplified block diagram of a regenerative repeater for unipolar NRZ signaling is shown in Fig 3-19 below:

![Block Diagram](image)

The amplifying filter increases the amplitude of the low-level input signal to a level that is compatible with the remaining circuitry and filters the signal in such a way as to minimize the effects of channel noise and ISI.

The bit synchronizer generates a cloak signal at the bit rate that is synchronized so that the amplified distorted signal is sampled at a point where the eye opening is at a maximum.

Comparator produces a high value when the sample value hold is larger than the threshold $V_T$. Generally $V_T$ is selected to be one-half the expected peak-to-peak variation of the sample values.

The overall probability of bit error is approximated by:

$$P_{me} \approx mP_e$$

where, $P_e$ is the probability of bit error for a single repeater.
Bit Synchronization

Synchronization signals are clock-type signals that are necessary within a receiver for detection of the data from the corrupted input signal. These clock-signals have a precise frequency and phase relationship with respect to the received input signal, and they are delayed when compared to the clock signals at the transmitter since there is a propagation delay in the channel.

Digital communications need at least three types of synchronization signals:

1. bit sync : to distinguish one bit interval from another
2. frame sync : to distinguish group of data
3. carrier sync : for band-pass signaling with coherent detection

We will concentrate on systems with bit synchronizers that derive the sync directly from the corrupted signal because it is often not economically feasible to send sync over a separate channel.

For example, the bit synchronizer for a unipolar RZ code with a sufficient number of alternating binary 1’s and 0’s is almost trivial since the PSD of that code has a delta function at the bit rate \( f = R \).

Therefore the sync clock signal can be obtained by passing the received unipolar RZ waveform through a narrowband band-pass filter that is tuned to \( f_0 = R = 1/T_b \).

This is shown in Fig. 3-20 below. The device is a square-law bit synchronizer for polar NRZ signaling.
Alternatively a phase-locked-loop (PLL) can be used to extract the sync signal from the unipolar RZ line by locking the PLL to the discrete spectral line at $f = R$. 

Figure 3-20  Square-law bit synchronizer for polar NRZ signaling.
For a polar NRZ line code, the bit synchronizer is slightly more complicated. Here the filtered \textit{polar NRZ} line code is converted to a \textit{unipolar RZ} waveform by using a \textit{sqaue-law circuit} (full-wave rectifier). The clock signal is easily recovered by using a filter or a PLL since the unipolar RZ code has a delta function in its spectrum at $f = R$.

An other technique utilizes the symmetry property of the line code itself.

A properly filtered line code has a pulse shape that is symmetrical about the optimum clocking (sampling) time, provided that the data are alternating between 1’s and 0’s. Fig. 3-21 shows such an early-late bitsyncronizer for polar NRZ signaling.
Power Spectra for Multilevel Signals

Multilevel signaling provides reduced bandwidth when compared with binary signaling. To reduce the signaling bandwidth Fig. 3-22 shows how a binary signal is converted to a multilevel signal where an $l$-bit DAC is used to convert the binary signal with data rate $R$ bits/sec to an $L = 2^l$ level multilevel digital signal.
For example assume $l = 3$ bit DAC is used, so that $L = 8$ levels.
$T_s$ is the time it takes to send one multilevel symbol.

From the figure we see that

$$D = \frac{1}{T_s} = \frac{1}{3T_b} = \frac{R}{3}$$

Or in general the baud rate is

$$D = \frac{R}{l}$$

Now let us look at PSD of multilevel signals:

PSD can be obtained by using equation 3-36a of the book.

$$P_s(f) = \frac{|F(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{j2\pi kf/T_s} \quad \text{eq.}(3-36a)$$

Evaluating $R(k)$ for $k = 0 \rightarrow R(0) = 21$

Since $P_s = \frac{1}{8}$ $k \neq 0 \rightarrow R(0) = 0$

Then

$$P_{w2}(f) = \frac{|F(f)|^2}{T_s} (21 + 0)$$

Where the pulse width (symbol width) is now $T_s = 3T_b$.

For rectangular pulses the PSD is:

$$P_{w2}(f) = 63T_b \left( \frac{\sin 3\pi f T_b}{3\pi f T_b} \right)^2$$
In general, for the case of $L = 2^l$ levels the PSD of a multilevel signal with rectangular pulse shapes is:

$$P_{\text{multilevel NRZ}}(f) = K \left( \frac{\sin(l\pi f T_b)}{l\pi f T_b} \right)^2$$

**Inter Symbol Interference**

If flat-top multilevel pulses are filtered improperly they will spread in time and the pulse for each symbol may be smeared into adjacent time slots and cause what is known as inter symbol interference (ISI).

![Figure 3-23 Examples of ISI on received pulses in a binary communication system.](image-url)
The important thing is to be able to restrict the bandwidth and yet not introduce ISI. Pulse shaping has to be used.