SPREADING CODES IN CDMA DETECTION

by

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Abstract

This study deals with the main characteristics of the Maximal Length, Gold, Kasami, Walsh and variable-length orthogonal codes and their functions in code-division multiple access (CDMA) networks.

The important properties of the sequences are examined. A method of multiple spreading for channelization and scrambling in CDMA is described. Auto and cross correlation properties of some codes are mentioned according to obtained graphs. Also, importance of spreading codes on CDMA detection is mentioned.

The main focus of this study is to emphasize the importance of code properties for detection. In other words, the effect of codes on DS-CDMA performance.
Chapter 1

Introduction

Direct Sequence Code Division Multiple Access (DS-CDMA) is a method of multiplexing users by distinct codes and in this method all users use the same bandwidth. In DS-CDMA, each user has its own spreading code. The selection of a good code is important, because auto-correlation properties and length of the code sets bound on the system capacity.

The code sets can be divided into two classes: orthogonal codes and non-orthogonal codes. Walsh sequences fall in the first category, while the other sequences (PN, Gold and Kasami) are shift-register sequences. When the user codes are orthogonal, the output of the correlator in the receiver is zero except the desired sequence.

In synchronous DS-CDMA systems the code sequence in the receiver is exactly same with that in the transmitter, so there is no time shift between the users. When the user codes are orthogonal in the synchronized systems, there is no multiple access interference between the users after despreading. In practice, it is difficult to realize synchronized DS-CDMA systems and time shifts between users decrease the systems capacity.

The most important measures that specify the codes performance are, the orderly low values of cross-correlation between codes and the rate of effect of auto-correlation values from time shifts.

In this study, fundamental properties of auto and cross-correlation of some important codes have been examined in case of asynchronous situation. Also the effect of code properties on DS-CDMA systems analysis and multiple spreading technique for channelization and scrambling in CDMA and are focused. Finally, in the last section, the importance of codes for the CDMA performance is discussed.

1.1 Problem Definition

In multi-user systems, the main reason that affects the performance is the multiple access interference. Especially in an occasion where users are mobile, asynchronous and power imbalance problems emerges among the users. Because of this reason, the selection of spreading codes to differentiate the users plays an important role in the system capacity.
In the DS-CDMA technique, each bits of the users data are multiplied with a code in the transmitter. The code sequence used in transmitter performs the role of spreading code. The baseband model of a DS-CDMA system is shown in figure 1.1. Let $b_k$ denote a binary data sequence, and $c_k$ denote a code sequence. The waveforms $b(t)$ and $c(t)$ denote polar representations in terms of two levels as ±1. By multiplying the information bit by the code, each information bit is divided into a small time increments that are called chips. The received data $r(t)$ consist of transmitted signal $m(t)$ plus an interference denoted by $i(t)$. To recover the original signal $b(t)$, the received signal $r(t)$ is multiplied with the code sequence that used in transmitter then passed through the low-pass filter. Finally a decision is made by the receiver. As an example, we can consider an

![Diagram](image)

Figure 1.1: The baseband model of a DS-CDMA system.

simple CDMA system by ignoring the channel. Let us think the first bits of four users. By multiplying each bit with a (PN)code, users bits are represented by seven chips(spreading) as shown below.

$\begin{align*}
U_1[1]: & 1 & C_1: & -1 -1 1 1 1 -1 1 \Rightarrow & -1 -1 1 1 1 -1 1 \\
U_2[1]: & -1 & C_2: & 1 -1 -1 1 1 -1 \Rightarrow & -1 1 1 -1 -1 1 1 \\
U_3[1]: & 1 & C_3: & -1 1 -1 -1 1 1 \Rightarrow & -1 1 -1 -1 1 1 1 \\
U_4[1]: & -1 & C_4: & 1 -1 1 -1 1 1 \Rightarrow & -1 1 -1 1 1 -1 1 \\
\end{align*}$

$+ r[1]: -4 2 0 0 2 -2 2 \rightarrow \text{received data}$
The received data $r$ consists information of four users. To recover the original bits of users from the received data, the received data should multiplied with the code sequence in the receiver that is exactly same with that is used for spreading the original data in transmitter (despreading). We assume that the receiver operates in perfect synchronism with the transmitter. As a last step, decision is made by comparing the results with a threshold value as shown below.

$$r[1] * C_1 = 4 \cdot 2 + 2 + 2 + 2 = 8 > 0 \Rightarrow 1$$
$$r[1] * C_2 = -4 \cdot 2 + 2 - 2 - 2 = 8 < 0 \Rightarrow -1$$
$$r[1] * C_3 = 4 + 2 + 2 + 2 = 8 > 0 \Rightarrow 1$$
$$r[1] * C_4 = -4 - 2 - 2 + 2 = 8 < 0 \Rightarrow -1$$

As shown from the example above, in recovering the original data, the selection of codes and their correlation with each other are of significant importance. Also perfect synchronism plays a very important role in the system performance. In this study, different spreading codes have been examined as time shifts, auto-correlation and cross-correlation functions points of view. The relation of spreading codes with the direct sequence code-division multiple access (DS-CDMA) analysis has been researched by taking into consideration the asynchronous case.
Chapter 2
Spreading Codes

In DS-CDMA system, for despreading operation, the received data should multiplied with the same code in the receiver. So the other user codes in the same frequency band must be uncorrelated with the desired user code. For this reason the DS-CDMA codes have to be designed so as to possess very low cross-correlation.

Auto-correlation shows the measure of similarity between the code and it’s cyclic shifted copy. Because of this reason, the codes that have the best properties of auto-correlation have frequently been used in removing the asynchronous in communication systems. The auto-correlation can be expressed as below[1].

\[ R_c(k) = \sum_{n=1}^{N} a_n a_{n+k} \]  

(2.1)

Cross-correlation is the measure of similarity between two different codes. In other words cross-correlation describes the interference between codes \(a_n\) and \(b_n\)[1].

\[ R_c(k) = \sum_{n=1}^{N} a_n b_{n+k} \]  

(2.2)

where \(a_n\) and \(b_n\) are the elements of two different codes and have period N.

2.1 PSEUDONOISE(PN) SEQUENCE

A pseudonoise(PN) sequence is a binary sequence of 1’s and 0’s and it is periodic. It’s some characteristics that are similar to random binary sequences(having equal # of 0’s and 1’s),very low correlation between any two shifted version of the sequence and low cross-correlation between any two sequences.
Pseudo-Random sequence is not random(deterministic) but it looks randomly for the user who doesn’t know the code. The larger the period of the PN spreading code, will be more random binary wave and it is harder to detect it.
A PN sequence is generated by a feedback shift register which is diagrammed in Fig 2.1. It consist of a shift register made up of m flip-flops and a logic circuit. The flip-flops in the shift register are regulated by a single timing clock. Binary sequences are shifted through the shift registers and the output of the various
stages are logically combined and feedback as the input to the first stage. The initial contents of the flip-flops determine the contents of the memory. The generated PN sequence is determined by mainly three factors that are length \( m \) of the shift register, flip-flop’s initial state and the feedback logic.

The number of possible states of the shift register is at most \( 2^m \) for a \( m \) flip-flops. So the generated PN sequence must become periodic with a period of at most \( 2^m \).

When the feedback logic consists of exclusive-OR gates, the shift register is called a linear and in such a case, the zero state is not permitted. Therefore the period of a PN sequence produced by a linear \( m \)-stage shift register can not exceed \( 2^m - 1 \). When a sequence of period \( 2^m - 1 \) generated, it is called a maximal-length (ML) sequence.

Maximal-length sequences satisfy the following three properties.

**Balance Property:** In each period of maximum-length sequence, the number of 1s is always one more than the number of 0s. So there must be \( 2^{m-1} \) ones and \( 2^{m-1} - 1 \) zeros in a full period of the sequence.

**Run Property:** Here, the 'run' represents a subsequence of identical symbols (1’s or 0’s) within one period of the sequence. The length of this subsequence is the length of the run. Among the runs of 1’s and 0’s in each period of a maximum-length sequence, one-half the run of each kind are of length one, one-fourth are length two, one-eighth are of length three, etc. For a maximum-length sequence generated by a linear feedback shift register of length \( m \), the total number of runs is \((N+1)/2\) where \( N=2^m - 1 \).

**Correlation Property:** The autocorrelation function of a maximum-length sequence is periodic, binary valued and has a period \( T=NT_c \) where \( T_c \) is chip duration. This property is called the correlation property. The autocorrelation function is\([2]\)

\[
R_c(\tau) = \begin{cases} 
1 - \frac{N+1}{NT_c} |\tau|, & |\tau| < \frac{N}{T_c} \\
\frac{1}{N}, & \text{for the remainder of the period}
\end{cases}
\]
Figure 2.2: Autocorrelation function for a maximum-length sequence with chip duration $T_c$ and period $NT_c$.

which is shown in figure 2.2. Since the correlation between different shifts of an m-sequence is almost zero, they can be used as different codes with an excellent correlation property.

As it is mentioned before, a maximum-length sequence can be generated by using a linear feedback shift register that was shown in figure 2.1. The feedback logic for a desired period N can be found from the tables of the necessary feedback connections for varying number of flip-flops. The first number of a feedback tap tells the length of the shift register, in other words the number of flip-flops and the rest of the numbers tell us the location of exclusive-OR gates.

Welch obtained the following lower bound on the cross-correlation between any pair of binary sequences of period L in a set of N sequences[1]:

$$R_c(k) \geq L \sqrt{\frac{N-1}{NL-1}} \equiv \sqrt{L} \quad (2.3)$$

2.2 GOLD SEQUENCES

The autocorrelation properties of m-sequence can not be bettered but they don’t exhibit a good cross-correlation properties for CDMA. It’s know that, a set of spreading codes used for multiple access system should have as little mutual interference as possible. For this reason, a particular class of PN sequences are used that are called Gold sequences. They can be chosen such that, the cross-correlation values between the codes over a set of codes are uniform and bounded.

Gold codes can be generated by modulo-2 addition of two maximum-length sequences with the same length. The code sequences are added chip by chip by synchronous clocking. The generated codes are of the same length as the two m-sequences which are added together.
One of the advantage of Gold codes is in generating large number of codes. To define a set of Gold codes, preferred pairs of m-sequences are used. If a is an m-sequence of length N, the second sequence a’ can be obtained by sampling every qth symbol of a. The second sequence is called decimation of the first sequence.

The following conditions are sufficient to define a preferred pair a and a’ of m-sequences:

1. \( n \neq 0 \); that is, \( n \) is odd or \( n=2 \)
2. \( a’ = a[q] \) where \( q \) is odd and either
   \[ q = 2^k + 1 \quad \text{or} \quad q = 2^{2k} - 2^k + 1 \]
3. \( gcd(n, k) = \begin{cases} 1, & \text{for } n \text{ odd;} \\ 2, & \text{for } n=2. \end{cases} \)

The decimation of an m-sequence may or may not yield another m-sequence. The set of Gold codes for this preferred pair of m-sequence is defined by \( \{ a, a’, a+a, a+Da’, a+Da'^2, \ldots, a+DN^{-1}a’ \} \) where D is the delay element. An illustration of generating a Gold set is shown in figure 2.3[1]. The \( 2^5+1=33 \) codes are generated by the above structure as follows:

Sequence 1: 111110001101101000100101100
Sequence 2: 1111100100110000101101010001110
0 shift combination: 0000000111101111111101100010
1 shift* combination: 0001010101111000010100001100
\ldots
30 shift combination: 100010001000101000110001101011
*shift of sequence 2 to the left

The N+1 elements of a Gold codes sets have the property that any pair of

\[ 0 2 \]
\[ 1 \]
\[ 2 \]
\[ 3 \]
\[ 4 \]
\[ 5 \]

Figure 2.3: Generation of a Gold code set.
codes in the set have a three-valued cross-correlation. In this set, except the sequences $a$ and $a'$, the rest of the sequences are not m-sequences. Hence, their autocorrelation functions are not two-valued, and it takes the same three values as cross-correlation. The three-level cross-correlation values of Gold sequences are as given in table 1\[2\].

<table>
<thead>
<tr>
<th>Shift-Register Length, $m$</th>
<th>Period (CodeLength)</th>
<th>Cross-Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ odd</td>
<td>$N=2^m - 1$</td>
<td>$-1/N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-(2^{(m+1)/2} + 1)/N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2^{(m+1)/2} - 1)/N$</td>
</tr>
<tr>
<td>$m$ even and not divisible by 4</td>
<td>$N=2^m - 1$</td>
<td>$-1/N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-(2^{(m+2)/2} + 1)/N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2^{(m+2)/2} - 1)/N$</td>
</tr>
</tbody>
</table>

Table 1. Three-Level Cross-Correlation Properties of Gold Sequences

2.3 KASAMI SEQUENCES

Kasami sequence sets are one of the important types of binary sequence sets because of their very low cross-correlation. Kasami codes are based on PN codes of length $N=2^m-1$ where $m$ is an even integer.

![Figure 2.4: Generation of Kasami set](image)

There are two different sets of Kasami sequences. Generation of a small set of Kasami sequences is similar to the generation of Gold sequences with $M=2^n/2$ binary sequences of period $N=2^n-1$, where $n$ is even. To generate a Kasami sequence, first of all, the sequence $a'$ is found by selecting every $(2^n/2+1)^{st}$ bit of an m-sequence $a$. The resulting sequence $a'$ is an m-sequence. The first Kasami
sequence can be found by adding (modulo-2 addition) the sequences \(a\) and \(a'\). Then by adding all \(2^{n/2}-2\) cyclic shifts of the sequence \(a'\) with the sequence \(a\), a new set of Kasami sequences can be formed. By including the sequence \(a\) in the set, a set of \(2^{n/2}\) sequences can be found. For example, for the case of \(m=4\), we take 15 length PN code and take it’s every 5th bit and keep repeating it to find the sequence \(a'\). The first member of the set is found by adding \(a'\) with the PN code \(a\) that is shown in figure 2.4. We then, shift the Kasami code by 1 bit and produce another member of the set.

The large set of Kasami sequences contains both the Gold sequences and the small set of Kasami sequences as subsets.

### 2.4 HADAMARD-WALSH(ORTHOGONAL) CODES

The Hadamard-Walsh codes are generated in a set of \(N=2^n\) codes with length \(N=2^n\). The generating algorithm is as follows [1].

\[
H_{2N} = \begin{bmatrix}
H_N & H_N \\
\overset{H}{H}_N & \overset{\overset{\text{Binary complement}}{\text{over underscore}}}{}H_N
\end{bmatrix}
\]

where \(N\) is a power of 2 and over underscore denotes the binary complement of the bits in the matrix. The smallest set of \(N=0\) is \(H_0 = [1]\) with the length 1. The rows or columns of matrix \(H_N\) are the Hadamard-Walsh codes since the matrix \(H_N\) is symmetric. The sets of 2 and 4 codes are shown below.

\[
H_2 = \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix}
\]

\[
H_4 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1
\end{bmatrix}
\]

As shown above, in each set, the first row of the matrix consist all 1’s and rest of the rows contains \(N/2\) 0’s and \(N/2\) 1’s. Also row \(N/2\) starts with \(N/2\) 1’s and ends with \(N/2\) 0’s.

Orthogonality is the most important property of Hadamard-Walsh codes. Because of this orthogonality property, the cross-correlation between any two Hadamard-Walsh codes of the same set (matrix) is zero, when system is perfectly synchronized.

Walsh codes are not maximal length or PN type codes for spread spectrum. Although the members of the set are orthogonal, they do not give any spreading. They are used in forward channel of IS-95 CDMA type system for their orthogonality.

Walsh code spreading can be used if all users of the same channel are synchronized in time, because the cross-correlation between different shifts of Walsh codes is not zero.
2.5 VARIABLE-LENGTH ORTHOGONAL CODES

Variable-length orthogonal codes are designed to improve the capability of the system by using higher bit rates. Depending on the desired bit rates and spreading bandwidth in the system, a range for the code length can be obtained.

Variable length orthogonal codes are generated by using tree-structure as shown below[3]. Starting from $C_1=1$, a set of $2^K$ spreading codes can be generated at the $k$th layer ($k=1,2,...,K$) from the root of the tree. The code length of the $k$th layer is $2^K$ chips. The generated codes of the same layer from a same layer, form a set of Walsh codes and they are orthogonal.

Denoting the set of $N$ binary spreading codes of $N$-chip length by $N \times N$ size of matrix $C_N$, it can be expressed as shown below[3].

$$C_N = \begin{bmatrix}
C_N(1) \\
C_N(2) \\
C_N(3) \\
\vdots \\
C_N(N-1) \\
C_N(N)
\end{bmatrix} = \begin{bmatrix}
C_{N/2}(1)C_{N/2}(1) \\
C_{N/2}(1)C_{N/2}(2) \\
C_{N/2}(2)C_{N/2}(1) \\
C_{N/2}(2)C_{N/2}(2) \\
\vdots \\
C_{N/2}(N/2)C_{N/2}(N/2) \\
C_{N/2}(N/2)C_{N/2}(N/2)
\end{bmatrix}$$

where $C_N(n)$ is the row vector of $N$ elements and $N=2^K$ ($K$ is a positive integer). $\overline{C_{N/2}(n)}$ is the binary complement of $C_{N/2}(n)$ and is the row vector of $N/2$ elements.
Any two codes of different layers shown in fig 2.5 are orthogonal except for the case that one of the two codes is a mother of the other. For example all of $C_{32}(2)$, $C_{16}(1)$, $C_{8}(1)$, $C_{4}(1)$ and $C_{2}(1)$ are mother codes of $C_{64}(3)$ so they are not orthogonal against $C_{64}(3)$. Furthermore, if a code of any layer is assigned to a user, all the codes generated from this code can not be assigned to other users of the same bandwidth requesting lower rates. This is restriction in order to maintain orthogonality.

### 2.6 MULTIPLE SPREADING CODE ALLOCATION

As it is known, all users in a CDMA systems are multiplied by a code sequence that has a chip rate is greater than the data rate. The way to orthogonalize the users is, multiplying each user’s binary input by a short spread sequence which is orthogonal to all other user of the same cell. The short orthogonal codes are called channelization codes. One type of this binary orthogonal sequences is the variable-length orthogonal codes that is explained in previous section.

After distinguishing the users of the same cell by using channelization codes, the users of different cells are distinguished by multiplication of data with a long pseudorandom sequence. The long PN sequences are called scrambling codes. Hence, each transmission channelization code is distinguished by a scrambling code.

![Figure 2.6: Application of Walsh and PN codes in the forward link of CDMA](attachment:image)

The application of different spreading in forward and reverse links of CDMA(IS-95) is shown in figures 2.6 and 2.7. As it can be seen from the figure, in the forward link, orthogonal(Walsh) spreading is used because all users are synchronized, where the code channels are distinguished by different short spreading
codes. Conversely, for the reverse link all users are asynchronous and therefore, channelization codes can be implemented using variable-length orthogonal codes. Also, in the forward link, all the base stations of different cells use the same PN sequence as the scrambling code but in the reverse link, each base station is identified by a unique time offset of its pseudorandom binary sequence.

Figure 2.7: Application of Walsh and PN codes in the reverse link of CDMA
Chapter 3

Simulation Results

In the third chapter, the fundamental properties and generation of some codes are discussed. Also it was mentioned before that auto and cross-correlation of these codes play a very important role in the CDMA systems performance. Hence, in this chapter, these properties are examined and compared according to the simulation results, in case of asynchronous situation.

As it can be seen from figure 3.1(a), the auto-correlation properties of PN and Gold codes are exactly same. These properties are very similar to the orthogonality properties since the correlation values between codes for the time shifts which are less than 1 chip are very low ($7.8 \times 10^{-3}$).

In figure 3.1(b), the cross-correlation values are shown for the interval $[-2T_c, 2T_c]$. It is clear that while the cross-correlation values are bounded by three values, PN codes have higher and multi values. Since the PN codes are chosen from the same set, the cross correlation properties are similar with the shifted autocorrelation properties. Time shifts less than one chip are modelled by increasing the resolution four times. In the new generation of DS-CDMA systems, Gold codes are preferred since cross-correlation properties are required. Especially for the case of asynchronous, the cross-correlation values of PN codes are high which cause multiple access interference (MAI).

In figure 3.2, the correlation values of PN and Gold sequences are shown for the case of higher time shifts $[-10T_c, 10T_c]$ (PN codes are chosen from the same set). The highest cross-correlation value of Gold code is 0.134.

In figure 3.3, the cross-correlation of two PN sequences are shown which are chosen from two different sets that are produced by different feedback connections. In the IS-95 standards, DS-CDMA systems, a long code is produced ($N=2^{42}-1$) and the parts of this code are used as spreading codes for different users. In figure 3.4, auto and cross-correlation values of Kasami sequences are shown. It is clear that, they have higher auto-correlation values for the time shifts which are greater than one chip as compare to PN and Gold sequences. It’s maximum cross-correlation value is around 0.333 (absolute value).

Mathematically it is proven that, for the large L and m odd, the maximum value of the cross-correlation function between any pair of Gold sequences is $R_{max}=\sqrt{2L}$, for even $R_{max}=2\sqrt{L}$. For the Kasami sequences, maximum cross-correlation value is found as $R_{max}=\sqrt{L}$. 

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Figure 3.1: Correlation values of [7 1]PN-code and [7 3]-[7 3 2 1]Gold code. N=127. Initial contents of flip-flops: 10000000

Given a set of N sequences of period L, a lower bound on their maximum cross-correlation is

$$R_{\text{max}} \geq L \sqrt{\frac{N - 1}{NL - 1}}$$  \hspace{1cm} (3.1)

which for one large values of L and N, is approximated as $$R_{\text{max}} \geq \sqrt{L}$$. Comparing this lower bound with the maximum value of the cross-correlation function between any pair of Gold sequences, it is clear that Gold sequences are slightly suboptimal. On the other hand, it is observed that Kasami sequences satisfy the lower bound for large values of L.

Because of all these properties, we can conclude that Gold & Kasami sequences are appropriate for CDMA applications.
Figure 3.2: Correlation values of [7 1]PN-code and [7 3]-[7 3 2 1]Gold code. N=127. Initial contents of flip-flops:1000000

Figure 3.3: Correlation values of PN-codes chosen from the sets of [7 1] and [7 6 5 4]. N=127. Initial contents of flip-flops:1000000
Figure 3.4: Correlation values of Kasami codes. N=127. Initial contents of flip-flops:1111
Chapter 4

Conclusion

The probability of error of a desired user in the direct sequence spread spectrum system with K multiple access users can be defined as

$$P(E) = f(P_k, \phi_k, \Delta_k)$$

where it is a function of power of the $k$th user, $P_k$, phase shift of the $k$th user caused by modulation, $\phi_k$, and the amount of shift of $k$th user caused by the asynchronous system, $\Delta_k$. The received signal will consist of sum of $K$ different transmitted data (one is desired user and $K-1$ undesired users). Reception is achieved by correlating the received data with the desired code sequence to produce a decision variable.

When the interference exists in the system, the probability of error is Q-function that is function of signal to interference noise ratio (SINR). For the case of no interference, Q-function is a function of signal to noise ratio (SNR). The average bit error probability for the case of interference existence and synchronized system is given as below [5].

$$P_e = Q\left(\sqrt{\frac{1}{3N} \sum_{k=1}^{K-1} \frac{P_k}{P_0} + \frac{N_0}{Nc T_b P_0}}\right) \quad (4.1)$$

where $P_0$ is the power of desired user, $T_b$ is the bit duration, $T_c$ is the chip duration, $N$ is the # of chip and $N_0$ is thermal noise. In some mobile radio environments, communication links are interference-limited. For the interference limited case (only MAI, without noise), the average bit error probability is given as [5]

$$P_e = Q\left(\sqrt{\frac{3N}{\sum_{k=1}^{K-1} \frac{P_k}{P_0}}}\right) \quad (4.2)$$

In the noninterference limited case, for perfect power control, which means power of all users are same where $p_k = P_0$ for all $k=1$...K-1 [5],

$$P_e = Q\left(\sqrt{\frac{1}{3N} + \frac{N_0}{2T_c P_0}}\right) \quad (4.3)$$

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where $K$ is the number of users. The interference limited case with perfect power control, the equation (4.3) can be approximated by

$$P_e = Q \left( \sqrt{\frac{3N}{K-1}} \right)$$

(4.4)

As it can be seen from the equation (4.4), as the number of users increase the probability of error increases as well, so they are directly proportional. As a result, multiple access interference that is caused by the undesired users is directly related with the cross-correlation properties of the codes of these users so, since the probability of error depends on the MAI, the effect of codes and their correlation properties play a very important role in the detection.
Chapter 5

References


