DC Circuits:

Capacitors and Inductors

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Capacitors and Inductors: Introduction

- Introduction
- Capacitors
- Series and Parallel Capacitors
- Inductors
- Series and Parallel Inductors
Capacitors and Inductors: Introduction

- **Resistors** are passive elements which dissipate energy only.
- **Two important passive linear circuit elements:**
  1. Capacitor
  2. Inductor
- Capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time.
- Capacitors and inductors are called storage elements.
Capacitors and Inductors

MICHAEL FARADAY (1791–1867)

English chemist and physicist, discovered electromagnetic induction in 1831 which was a breakthrough in engineering because it provided a way of generating electricity. The unit of capacitance, the farad, was named in his honor.
Capacitors and Inductors

- A capacitor consists of two conducting plates separated by an insulator (or dielectric).

Plates may be aluminum foil while the dielectric may be air, ceramic, paper, or mica.
Capacitors and Inductors

• Three factors affecting the value of capacitance:

\[ C = \frac{\varepsilon A}{d} \]

1. **Area** \((A)\): the larger the area, the greater the capacitance.
2. **Spacing between the plates** \((d)\): the smaller the spacing, the greater the capacitance.
3. **Material permittivity** \((\varepsilon)\): the higher the permittivity, the greater the capacitance.
Capacitors and Inductors

- The relation between the charge in plates and the voltage across a capacitor is given below.

\[ q = Cv \]

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

\[ 1 \text{F} = 1 \text{C/V} \]

(1 farad = 1 coulomb/volt)
Capacitors and Inductors

Circuit symbols for capacitors: (a) fixed capacitors, (b) variable capacitors.

(a) Polyester capacitor, (b) Ceramic capacitor, (c) Electrolytic capacitor

(a-b) variable capacitors
Capacitors and Inductors

• Voltage Limit on a Capacitor:

According to: $q = Cv$

- The plate charge increases as the voltage increases. Also, the electric field intensity between two plates increases.

- If the voltage across the capacitor is so large that the field intensity is large enough to break down the insulation of the dielectric, the capacitor is out of work. Hence, every practical capacitor has a maximum limit on its operating voltage.
Capacitors and Inductors

- **Current**($i$)-**voltage**($v$) relationship of a Capacitor:

  \[ q = Cv \]

  Taking the derivative of both sides

  \[ i = C \frac{dv}{dt} \]

  where \[ i = \frac{dq}{dt} \]

  ![Diagram of a capacitor with current and voltage]
Capacitors and Inductors

**Physical Meaning:**

- When \( v \) is a constant voltage, then \( i = 0 \); a constant voltage across a capacitor creates no current through the capacitor, the capacitor in this case is the same as an open circuit.
- If \( v \) is abruptly changed, then the current will have an infinite value that is practically impossible. Hence, a capacitor is impossible to have an abrupt change in its voltage except an infinite current is applied.
Capacitors and Inductors

• **Important Properties of capacitors:**

1) A capacitor is an **open circuit to dc**.

2) The voltage on a capacitor **cannot change abruptly**.

![Voltage across a capacitor:](image)

(a) allowed, (b) not allowable; an abrupt change is not possible.

3) Ideal capacitor **does not dissipate energy**. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

4) A real, nonideal capacitor has a **parallel-model leakage resistance**. The leakage resistance may be as high as 100 MΩ and can be neglected for most practical applications. We will assume ideal capacitors.
Capacitors and Inductors

- **Important Properties of capacitors:**
- A capacitor has memory.

\[ i = C \frac{dv}{dt} \quad \Rightarrow \quad v = \frac{1}{C} \int_{-\infty}^{t} i \, dt \quad \Rightarrow \quad v = \frac{1}{C} \int_{t_0}^{t} i \, dt + v(t_0) \]

\[ v(t_0) = \frac{q(t_0)}{C} \] is the voltage across the capacitor at time \( t_0 \).

- Capacitor voltage depends on the past history \((-\infty \rightarrow t_0)\) of the capacitor current. Hence, the capacitor has memory.
Capacitors and Inductors

- **Important Properties of capacitors:**
- **Energy Storing in Capacitor**

  instantaneous power delivered to the capacitor is

  \[ p = vi = Cv \frac{dv}{dt} \]

  energy stored in the capacitor:

  \[
  w = \int_{-\infty}^{t} p \, dt = C \int_{-\infty}^{t} v \frac{dv}{dt} \, dt = C \left[ v(t) \right]_{v(-\infty)}^{v(\infty)} \cdot \frac{1}{2} Cv^2 \]

  \[ w = \frac{1}{2} Cv^2 \]

  note that \( v(-\infty) = 0 \),

  recall that \( q = Cv \)
Capacitors and Inductors

Ex. 6.1: (a) Calculate the charge stored on a 3-pF capacitor with 20 V across it. (b) Find the energy stored in the capacitor.

Solution

(a) since: \[ q = Cv, \]

\[ q = 3 \times 10^{-12} \times 20 = 60 \text{ pC} \]

(a) The energy stored:

\[ w = \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ} \]
Capacitors and Inductors

Ex. 6.2: The voltage across a 5- μF capacitor is: \( v(t) = 10 \cos 6000t \) V

Calculate the current through it.

Solution

(a) By definition current is:

\[
i = C \frac{dv}{dt}
\]

\[
i = C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt} (10 \cos 6000t)
\]

\[
= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \ A
\]
Capacitors and Inductors

Ex. 6.3: Determine the voltage across a 2-μF capacitor if the current through it is:

\[ i(t) = 6e^{-3000t} \text{mA} \]

Assume that the initial capacitor voltage is zero.

Solution

Since:

\[ v(t) = \frac{1}{C} \int_{0}^{t} i(\tau) d\tau + v(0) \quad \text{and} \quad v(0) = 0, \]

\[ v(t) = \frac{1}{2 \times 10^{-6}} \int_{0}^{t} 6e^{-3000\tau} \cdot 10^{-3} \, d\tau = \frac{3 \times 10^{3}}{-3000} e^{-3000t} \bigg|_{0}^{t} \]

\[ = (1 - e^{-3000t}) \text{V} \]
Capacitors and Inductors

Ex. 6.4: Determine the current through a 200- \( \mu \)F capacitor whose voltage is shown in Fig 6.9.

Solution

The voltage waveform can be described mathematically as:

\[
v(t) = \begin{cases} 
50t \text{ V} & 0 < t < 1 \\
100 - 50t \text{ V} & 1 < t < 3 \\
-200 + 50t \text{ V} & 3 < t < 4 \\
0 & \text{otherwise}
\end{cases}
\]
Capacitors and Inductors

Ex. 6.4: Determine the current through a 200- μF capacitor whose voltage is shown in Fig 6.9.

Solution

The Since \( i = C \frac{dv}{dt} \) and \( C = 200 \) μF, we take the derivative of \( v(t) \) to obtain:

\[
i(t) = 200 \times 10^{-6} \begin{cases} 
50 & 0 < t < 1 \\
-50 & 1 < t < 3 \\
50 & 3 < t < 4 \\
0 & \text{otherwise}
\end{cases}
\]

Thus the current waveform is:
Capacitors and Inductors

Ex. 6.5: Obtain the energy stored in each capacitor in circuit below under dc condition.

Solution

Under dc condition, we replace each capacitor with an open circuit. By current division,

\[ i = \frac{3}{3 + 2 + 4} (6 \text{mA}) = 2 \text{mA} \]

\[ \therefore \, v_1 = 2000i = 4 \text{V}, \]
\[ \therefore \, w_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2} (2 \times 10^{-3})(4)^2 = 16 \text{ mJ} \]

\[ v_2 = 4000i = 8 \text{V} \]
\[ w_2 = \frac{1}{2} C_2 v_2^2 = \frac{1}{2} (4 \times 10^{-3})(8)^2 = 128 \text{ mJ} \]
Capacitors and Inductors

Series and Parallel Capacitors

Parallel-connected $N$ capacitors

Equivalent circuit for the parallel capacitors.

$$i = i_1 + i_2 + i_3 + \ldots + i_N$$
Capacitors and Inductors

Series and Parallel Capacitors

\[ i = i_1 + i_2 + i_3 + \ldots + i_N \]

\[ i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \ldots + C_N \frac{dv}{dt} \]

\[ = \left( \sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt} \]

\[ C_{eq} = C_1 + C_2 + C_3 + \ldots + C_N \]

- The equivalent capacitance of \( N \) parallel-connected capacitors is the sum of the individual capacitances.
Capacitors and Inductors

Series and Parallel Capacitors

Series-connected $N$ capacitors

Equivalent circuit for the series capacitors.

$$v(t) = v_1(t) + v_2(t) + \ldots + v_N(t)$$
Capacitors and Inductors

Series and Parallel Capacitors

\[ U = U_1 + U_2 + U_3 + \cdots + U_N \]

\[ v = \frac{1}{C_1} \int_{t_0}^{t} i(t) \, dt + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^{t} i(t) \, dt + v_2(t_0) + \cdots + \frac{1}{C_N} \int_{t_0}^{t} i(t) \, dt + v_N(t_0) \]

\[ = \left( \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \right) \int_{t_0}^{t} i(t) \, dt + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0) \]

\[ = \frac{1}{C_{eq}} \int_{t_0}^{t} i(t) \, dt + v(t_0) \]

- The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N} \]

For \( N=2 \):

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \]
Capacitors and Inductors

Series and Parallel Capacitors

• Summary:

These results enable us to look the capacitor in this way:

• $1/C$ has the equivalent effect as the resistance.

The equivalent capacitor of capacitors connected in parallel or series can be obtained via this point of view, so is the $\text{Y-}\Delta$ connection and its transformation.
Ex. 6.6: Find the equivalent capacitance seen between terminals a and b of the circuit in the figure below.
Ex. 6.6: Find the equivalent capacitance seen between terminals a and b of the circuit in the figure below.

Solution

- 20 μF and 5 μF capacitors are in series:
  \[ \frac{20 \times 5}{20 + 5} = 4 \mu F \]

- 4 μF capacitor is in parallel with the 6 μF and 20 μF capacitors
  \[ 4 + 6 + 20 = 30 \mu F \]

- 30 μF capacitor is in series with the 60 μF capacitor.
  \[ C_{eq} = \frac{30 \times 60}{30 + 60} \mu F = 20 \mu F \]
Capacitors and Inductors

Ex. 6.7: For the circuit given below, find the voltage across each capacitor.

Solution

\[
C_{eq} = \frac{1}{\frac{1}{(40+20)} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}
\]
Capacitors and Inductors

**Ex. 6.7:** For the circuit given below, find the voltage across each capacitor.

### Total charge

\[ q = C_{eq} v = 10 \times 10^{-3} \times 30 = 0.3 \text{C} \]

This is the charge on the 20 mF and 30 mF capacitors, because they are in series with the 30 V source. (A crude way to see this is to imagine that charge acts like current, since \( i = dq/dt \))

### Therefore,

\[ v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V}, \]

\[ v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V} \]
Capacitors and Inductors

Ex. 6.7: For the circuit given below, find the voltage across each capacitor.

Having determined $v_1$ and $v_2$, we can use KVL to determine $v_3$ by

$$v_3 = 30 - v_1 - v_2 = 5\text{V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage $v_3$ and their combined capacitance is $40+20=60\text{mF}$.

$$\therefore v_3 = \frac{q}{60\text{mF}} = \frac{0.3}{60 \times 10^{-3}} = 5\text{V}$$
Capacitors and **Inductors**

**JOSEPH HENRY (1797-1878)**

American physicist, discovered **inductance** and constructed an **electric motor**.
Capacitors and **Inductors**

- An inductor is made of a coil of conducting wire.

**Inductance** is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

\[ L = \frac{N^2 \mu A}{l} \]
Capacitors and **Inductors**

\[ L = \frac{N^2 \mu A}{l} \]

\[ \mu = \mu_r \mu_0 \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ (H/m)} \]

- **\( N \)**: number of turns.
- **\( l \)**: length.
- **\( A \)**: cross-sectional area.
- **\( \mu \)**: *permeability of the core*

(a) Solenoidal inductor, (b) toroidal inductor, (c) chip inductor.
Capacitors and **Inductors**

- **Circuit symbols for inductors:**

  (a) air-core, (b) iron-core, (c) variable iron-core.
Capacitors and Inductors

- **Voltage**($v$) - **current**($i$) relationship of an Inductor:

  $$v = L \frac{di}{dt}$$

  Integrating both sides gives:

  $$i = \frac{1}{L} \int_{-\infty}^{t} v(t) \, dt$$

  $$i(t_0)$$ is the total current for $-\infty < t < t_0$ and $i(-\infty) = 0$

  [Diagram showing the voltage-current relationship of an inductor with a slope of $L$.]
Capacitors and **Inductors**

- **Physical Meaning:**

  \[ v = L \frac{di}{dt} \]

- When the **current** through an inductor is a **constant**, then the voltage across the inductor is zero, same as a **short circuit**.

- No **abrupt change of the current** through an inductor is possible except an **infinite** voltage across the inductor is applied.

- The inductor can be used to generate a high voltage, for example, used as an igniting element.
Capacitors and **Inductors**

- **Important Properties of inductors:**

  1) An inductor is a short to dc.
  2) The current through an inductor cannot change abruptly.

![Current through an inductor: (a) allowed, (b) not allowable; an abrupt change is not possible.](image)

  3) Ideal inductor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.

  4) A practical, nonideal inductor has a significant resistive component. This resistance is called the winding resistance. The nonideal inductor also has a winding capacitance due to the capacitive coupling between the conducting coils. We will assume ideal inductors.
Capacitors and **Inductors**

- **Important Properties of inductors:**
- An inductor **has memory**.

\[ v = L \frac{di}{dt} \quad \Rightarrow \quad di = \frac{1}{L} v \, dt \quad \Rightarrow \quad i = \frac{1}{L} \int_{-\infty}^{t} v(t) \, dt \]

\[ i = \frac{1}{L} \int_{t_0}^{t} v(t) \, dt + i(t_0) \]

- **Inductor current depends on the past history** \((-\infty \rightarrow t_0)\) **of the inductor current**. Hence, the inductor **has memory**.

\[ v_i \]

\[ + \]

\[ L \]

\[ - \]
Capacitors and Inductors

• Important Properties of inductors:
• Energy Storing in Inductor

Power delivered to the inductor is

\[ p = vi = \left( L \frac{di}{dt} \right) i \]

Energy stored in the inductor:

\[ w = \int_{-\infty}^{t} p \, dt = \int_{-\infty}^{t} \left( L \frac{di}{dt} \right) i \, dt = L \int_{-\infty}^{t} i \, di = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty) \]

\[ w = \frac{1}{2} Li^2 \]

Note that \( i(-\infty) = 0 \).
Capacitors and Inductors

Ex. 6.8: The current through a 0.1-H inductor is \( i(t) = 10te^{-5t} \) A. Find the voltage across the inductor and the energy stored in it.

Solution

Since \( v = L \frac{di}{dt} \) and \( L = 0.1 \)H,

\[
v = 0.1 \frac{d}{dt} (10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t} (1 - 5t) \text{V}
\]

The energy stored is

\[
w = \frac{1}{2} Li^2 = \frac{1}{2} (0.1)100t^2 e^{-10t} = 5t^2 e^{-10t} \text{J}
\]
Capacitors and Inductors

Ex. 6.9: Find the current through a 5-H inductor if the voltage across it is

\[ v(t) = \begin{cases} 
30t^2, & t > 0 \\
0, & t < 0 
\end{cases} \]

Also find the energy stored within \( 0 < t < 5 \text{s} \). Assume \( i(0) = 0 \).

Solution

Since \( i = \frac{1}{L} \int_{t_0}^{t} v(t) \, dt + i(t_0) \) and \( L = 5 \text{H} \).

\[
i = \frac{1}{5} \int_{0}^{t} 30t^2 \, dt + 0 = \frac{1}{5} \cdot 6 \times \frac{t^3}{3} = 2t^3 \text{ A}
\]
Capacitors and Inductors

Ex. 6.9: Find the current through a 5-H inductor if the voltage across it is

\[ v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases} \]

Also find the energy stored within \( 0 < t < 5 \)s. Assume \( i(0)=0 \).

Solution

The power \( p = vi = 60t^5 \), and the energy stored is then

\[
w = \int p \, dt = \int_0^5 60t^5 \, dt = 60 \left. \frac{t^6}{6} \right|_0^5 = 156.25 \text{ kJ}
\]

Alternatively

\[
w(5) - w(0) = \frac{1}{2} Li^2(5) - \frac{1}{2} Li(0)
\]

\[
= \frac{1}{2} (5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}
\]
Ex. 6.10: Consider the circuit below in (a). Under dc conditions, find:
(a) $i$, $v_C$, and $i_L$.
(b) the energy stored in the capacitor and inductor.

Solution
Capacitors and Inductors

Ex. 6.10: Consider the circuit below in (a). Under dc conditions, find:

(a) $i$, $v_C$, and $i_L$.
(b) the energy stored in the capacitor and inductor.

Solution:

(a) Under dc condition: capacitor $\rightarrow$ open circuit
    inductor $\rightarrow$ short circuit

$$i = i_L = \frac{12}{1+5} = 2A, \quad v_c = 5i = 10V$$

(b) $$w_c = \frac{1}{2}Cv_c^2 = \frac{1}{2}(1)(10^2) = 50J,$$

$$w_L = \frac{1}{2}L_i^2 = \frac{1}{2}(2)(2^2) = 4J$$
Capacitors and Inductors

Series and Parallel Inductors

Series-connected $N$ inductors

Equivalent circuit for the series inductors:

$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N$$
Capacitors and **Inductors**

**Series and Parallel Inductors**

- **Applying KVL to the loop,**
  \[ v = v_1 + v_2 + v_3 + \ldots + v_N \]

- **Substituting** \( v_k = L_k \frac{di}{dt} \) **results in**
  \[ v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \ldots + L_N \frac{di}{dt} \]

  \[ = (L_1 + L_2 + L_3 + \ldots + L_N) \frac{di}{dt} \]

  \[ = \left( \sum_{K=1}^{N} L_K \right) \frac{di}{dt} = L_{eq} \frac{di}{dt} \]

\[ L_{eq} = L_1 + L_2 + L_3 + \ldots + L_N \]
Capacitors and Inductors

Series and Parallel Inductors

Paralleled-connected \( N \) inductors

Equivalent circuit for the parallel inductors.

\[
\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}
\]
Capacitors and Inductors

Series and Parallel Inductors

- Using KCL,

\[ i = i_1 + i_2 + i_3 + \ldots + i_N \]

- But

\[ i_k = \frac{1}{L_k} \int_{t_0}^{t} v dt + i_k(t_0) \]

\[ i = \frac{1}{L_k} \int_{t_0}^{t} v dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^{t} v dt + i_s(t_0) + \ldots + \frac{1}{L_N} \int_{t_0}^{t} v dt + i_N(t_0) \]

\[ = \left( \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N} \right) \int_{t_0}^{t} v dt + i_1(t_0) + i_s(t_0) + \ldots + i_N(t_0) \]

\[ = \left( \sum_{k=1}^{N} \frac{1}{L_k} \right) \int_{t_0}^{t} v dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v dt + i(t_0) \]

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N} \]
Capacitors and **Inductors**

**Series and Parallel Inductors**

- The *inductor* in various connection has the same effect as the *resistor*. Hence, the **Y-Δ** transformation of inductors can be similarly derived.
### Capacitors and Inductors

#### TABLE 6.1

<table>
<thead>
<tr>
<th>Relation</th>
<th>Resistor ($R$)</th>
<th>Capacitor ($C$)</th>
<th>Inductor ($L$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v-i$:</td>
<td>$v = iR$</td>
<td>$v = \frac{1}{C} \int_{t_0}^{t} i , dt + v(t_0)$</td>
<td>$v = \frac{L}{t} \frac{di}{dt}$</td>
</tr>
<tr>
<td>$i-v$:</td>
<td>$i = v/R$</td>
<td>$i = C \frac{dv}{dt}$</td>
<td>$i = \frac{1}{L} \int_{t_0}^{t} v , dt + i(t_0)$</td>
</tr>
<tr>
<td>$p$ or $w$:</td>
<td>$p = i^2 R = \frac{v^2}{R}$</td>
<td>$w = \frac{1}{2} C v^2$</td>
<td>$w = \frac{1}{2} L i^2$</td>
</tr>
<tr>
<td>Series:</td>
<td>$R_{eq} = R_1 + R_2$</td>
<td>$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$</td>
<td>$L_{eq} = L_1 + L_2$</td>
</tr>
<tr>
<td>Parallel:</td>
<td>$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$</td>
<td>$C_{eq} = C_1 + C_2$</td>
<td>$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$</td>
</tr>
<tr>
<td>At dc:</td>
<td>Same</td>
<td>Open circuit</td>
<td>Short circuit</td>
</tr>
</tbody>
</table>

Circuit variable that cannot change abruptly: Not applicable

$v$  \hspace{1cm} $i$

---

† Passive sign convention is assumed.
Capacitors and Inductors

Ex. 6.11: Find the equivalent inductance of the circuit given below.
Capacitors and Inductors

Ex. 6.11: Find the equivalent inductance of the circuit given below.

![Circuit Diagram]

Solution:

Series: \( 20\,\text{H}, 12\,\text{H}, 10\,\text{H} \rightarrow 42\,\text{H} \)

Parallel: \( \frac{7 \times 42}{7 + 42} = 6\,\text{H} \)

\[ \therefore L_{eq} = 4 + 6 + 8 = 18\,\text{H} \]
Capacitors and **Inductors**

**Ex:** Calculate the equivalent inductance for the inductive ladder network given in the circuit below.

![Inductive Ladder Network Diagram](image)
Capacitors and Inductors

**Ex 6.12:** For the circuit given below, \( i(t) = 4(2 - e^{-10t}) \text{mA} \).

If, \( i_2(0) = -1 \text{ mA} \),

Find:

(a) \( i_1(0) \)

(b) \( v(t), v_1(t), \) and \( v_2(t) \);

(c) \( i_1(t) \) and \( i_2(t) \)
Capacitors and Inductors

Ex 6.12: For the circuit given below, \( i(t) = 4(2 - e^{-10t}) \) mA.
If, \( i_2(0) = -1 \) mA,
Find: (a) \( i_1(0) \)
(b) \( v(t) \), \( v_1(t) \), and \( v_2(t) \);  
(c) \( i_1(t) \) and \( i_2(t) \)

Solution:

(a) \( i(t) = 4(2 - e^{-10t}) \) mA \( \rightarrow i(0) = 4(2 - 1) = 4 \) mA.
\[ \therefore i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{ mA} \]

(b) The equivalent inductance is \( L_{eq} = 2 + 4 \parallel 12 = 2 + 3 = 5 \text{ H} \)
\[ \therefore v(t) = L_{eq} \frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV} \]
\[ v_1(t) = 2 \frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV} \]
\[ \therefore v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{ mV} \]
Capacitors and Inductors

Ex 6.12: For the circuit given below, \( i(t) = 4(2 - e^{-10t}) \text{mA} \).

If, \( i_2(0) = -1 \text{ mA} \),

Find:
(a) \( i_1(0) \)
(b) \( v(t), v_1(t), \) and \( v_2(t) \);
(c) \( i_1(t) \) and \( i_2(t) \)

Solution:

\[
\begin{align*}
i_1(t) & = \frac{1}{4} \int_0^t v_2 \, dt + i_1(0) = \frac{120}{4} \int_0^t e^{-10t} \, dt + 5 \text{ mA} \\
& = -3e^{-10t} \bigg|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA} \\
i_2(t) & = \frac{1}{12} \int_0^t v_2 \, dt + i_2(0) = \frac{120}{12} \int_0^t e^{-10t} \, dt - 1 \text{ mA} \\
& = -e^{-10t} \bigg|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}
\end{align*}
\]

Note that \( i_1(t) + i_2(t) = i(t) \).