DC Circuits:

Methods of Analysis

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Nodal Analysis

• Steps to Determine **Node Voltages**:

1. Select a node as the **reference node**. Assign voltage $v_1$, $v_2$, $\ldots$, $v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the **reference node**.

2. Apply **KCL** to each of the $n-1$ non-reference nodes. Use Ohm’s law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.
Nodal Analysis

• Steps to Determine **Node Voltages**:

  Common symbols for indicating a reference node:
  (a) common ground, (b) ground, (c) chassis.
Nodal Analysis

• **Typical circuit for nodal analysis**

  (a) Given circuit, (c) voltages $v_1$ and $v_2$ are assigned with respect to the reference node (i.e., ground).
Nodal Analysis

- Typical circuit for nodal analysis:
  - Apply KCL to each non-reference node.

\[
I_1 = I_2 + i_1 + i_2 \\
I_2 + i_2 = i_3
\]

\[
i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}
\]

\[
i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1
\]

\[
i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2 (v_1 - v_2)
\]

\[
i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2
\]
Nodal Analysis

• Typical circuit for nodal analysis:
  ▪ Apply KCL to each nonreference node.

\[
I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}
\]

\[
I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}
\]

\[
I_1 - I_2 = G_1 v_1 + G_2 (v_1 - v_2)
\]

\[
I_2 = -G_2 (v_1 - v_2) + G_3 v_2
\]

\[
\begin{bmatrix}
G_1 + G_2 & -G_2 \\
-G_2 & G_2 + G_3
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
= 
\begin{bmatrix}
I_1 - I_2 \\
I_2
\end{bmatrix}
\]
Nodal Analysis
• **Example 3.1:** Calculate the *node voltages* in the circuit shown below.
Nodal Analysis

• Example 3.1: Calculate the node voltages in the circuit shown below.

• At node 1:

\[ i_1 = i_2 + i_3 \]

\[ 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \]

(Multiply each term by 4)

\[ \Rightarrow 20 = v_1 - v_2 + 2v_1 \]

\[ 3v_1 - v_2 = 20 \]
Nodal Analysis

- **Example 3.1**: Calculate the node voltages in the circuit shown below.

- **At node 2**:

\[ i_2 + i_4 = i_1 + i_5 \]
\[ i_2 + 10 = 5 + i \]
\[ 5 = i_5 - i_2 \]

\[ \Rightarrow 5 = \frac{v_2 - 0}{6} - \left( \frac{v_1 - v_2}{4} \right) \]
\[ 5 = \frac{v_2}{6} + \frac{v_2 - v_1}{4} \quad \text{(Multiply each term by 12)} \]

\[ 60 = 2v_2 + 3v_2 - 3v_1 \]
\[ -3v_1 + 5v_2 = 60 \]
Nodal Analysis

• Example 3.1: Calculate the node voltages in the circuit shown below.

\[ \text{node 1: } 3v_1 - v_2 = 20 \quad (1) \]

\[ \text{node 2: } -3v_1 + 5v_2 = 60 \quad (2) \]

**METHOD 1** Using the elimination technique, we add Eqs. (1) and (2).

\[ 4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V} \]

Substituting \( v_2 = 20 \) in Eq. (1) gives

\[ 3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V} \]
Nodal Analysis

• Example 3.1: Calculate the node voltages in the circuit shown below.

  node 1: \[ 3v_1 - v_2 = 20 \] (1)

  node 2: \[ -3v_1 + 5v_2 = 60 \] (2)

**Method 2** To use Cramer’s rule, we need to put Eqs. (1) and (2) in matrix form as

\[
\begin{bmatrix}
3 & -1 \\
-3 & 5
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
=
\begin{bmatrix}
20 \\
60
\end{bmatrix}
\] (3)

The determinant of the matrix is

\[
\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12
\]

We now obtain \( v_1 \) and \( v_2 \) as

\[
v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333 \text{ V}
\]

\[
v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}
\]
Nodal Analysis

• **Example**: Calculate the node voltages in the circuit shown below.
Nodal Analysis

• **Example 3.2**: Determine the voltages at nodes 1, 2 and 3.
Nodal Analysis

- **Example 3.2**: Determine the voltages at nodes 1, 2 and 3.
- **At node 1**:

\[ 3 = i_1 + i_x \]

\[ \Rightarrow 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \]

(Multiplying by 4 and rearranging terms)

\[ 3v_1 - 2v_2 - v_3 = 12 \]
Nodal Analysis

• **Example 3.2**: Determine the voltages at nodes 1, 2 and 3.

• **At node 2**:

\[ i_x = i_2 + i_3 \]

\[ \Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \]

(Multiplying by 8 and rearranging terms)

\[-4v_1 + 7v_2 - v_3 = 0\]
Nodal Analysis

- **Example 3.2**: Determine the voltages at nodes 1, 2 and 3.
- **At node 3**:

\[
i_1 + i_2 = 2i_x
\]

\[
\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}
\]

(Multiplying by 8 and rearranging terms)

\[
2v_1 - 3v_2 + v_3 = 0
\]
Nodal Analysis

- **Example 3.2**: Determine the voltages at nodes 1, 2 and 3.

**METHOD**

To use Cramer’s rule, we put Eqs. (1) to (3) in matrix form.

\[
\begin{bmatrix}
3 & -2 & -1 \\
-4 & 7 & -1 \\
2 & -3 & 1
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}
= 
\begin{bmatrix}
12 \\
0 \\
0
\end{bmatrix}
\]

From this, we obtain

\[
v_1 = \frac{\Delta_1}{\Delta}, \quad v_2 = \frac{\Delta_2}{\Delta}, \quad v_3 = \frac{\Delta_3}{\Delta}
\]

where \( \Delta, \Delta_1, \Delta_2, \) and \( \Delta_3 \) are the determinants to be calculated as follows. To calculate the determinant of a 3 by 3 matrix, we repeat the first two rows and cross multiply.

\[
\Delta = \begin{vmatrix}
3 & -2 & -1 \\
-4 & 7 & -1 \\
2 & -3 & 1
\end{vmatrix}
= 21 - 12 + 4 + 14 - 9 - 8 = 10
\]

Similarly, we obtain

\[
\Delta_1 = \begin{vmatrix}
12 & -2 & -1 \\
0 & 7 & -1 \\
12 & -3 & 1
\end{vmatrix}
= 84 + 0 + 0 - 0 - 36 - 0 = 48
\]
Nodal Analysis

• Example 3.2: Determine the voltages at nodes 1, 2 and 3.

\[ \Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 0 + 0 - 24 - 0 - 0 + 48 = 24 \]

\[ \Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = 0 + 144 + 0 - 168 - 0 - 0 = -24 \]

Thus, we find

\[ v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V} \]

\[ v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V} \]
Nodal Analysis: Cramer's Rule

Consider the linear system \( \begin{align*}
ax + by &= e \\
rx + dy &= f
\end{align*} \)
which in matrix format is
\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
e \\
f
\end{bmatrix}.
\]

Assume \( ad - bc \) nonzero. Then, \( x \) and \( y \) can be found with Cramer's rule as
\[
x = \frac{e \Delta_1}{\Delta} = \frac{\begin{vmatrix}
e & b \\
f & d
\end{vmatrix}}{\begin{vmatrix}
a & b \\
c & d
\end{vmatrix}} = \frac{ed - bf}{ad - bc},
\]
and
\[
y = \frac{f \Delta_2}{\Delta} = \frac{\begin{vmatrix}
a & e \\
c & f
\end{vmatrix}}{\begin{vmatrix}
a & b \\
c & d
\end{vmatrix}} = \frac{af - ec}{ad - bc}.
\]

The rules for 3x3 are similar. Given \( \begin{align*}
ax + by + cz &= j \\
dx + ey + fz &= k \\
gx + hy + iz &= l
\end{align*} \)
which in matrix format is
\[
\begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
j \\
k \\
l
\end{bmatrix}.
\]

Then the values of \( x, y \) and \( z \) can be found as follows:
\[
x = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix}
j & b & c \\
k & e & f \\
l & h & i
\end{vmatrix}}{\begin{vmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{vmatrix}}, \quad y = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix}
a & j & c \\
d & k & f \\
g & l & i
\end{vmatrix}}{\begin{vmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{vmatrix}}, \quad z = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix}
a & b & j \\
d & e & k \\
g & h & l
\end{vmatrix}}{\begin{vmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{vmatrix}}.
\]
Nodal Analysis with Voltage Sources

- **Case 1:** The voltage source is connected between a nonreference node and the reference node: The nonreference node voltage is equal to the magnitude of voltage source and the number of unknown nonreference nodes is reduced by one.

- **Case 2:** The voltage source is connected between two nonreference nodes: a generalized node (supernode) is formed.
Nodal Analysis with Voltage Sources

- A circuit with a supernode.
Nodal Analysis: Supernode

- A **supernode** is formed by **enclosing** a (dependent or independent) voltage source connected between two nonreference nodes and **any elements connected in parallel with it**.

- The required **two equations** for regulating the two nonreference node voltages are obtained by the KCL of the supernode and the **relationship of node voltages due to the voltage source**.
Nodal Analysis: Supernode

• Find the node voltages of the circuit below.

\[ 2 - 7 - i_1 - i_2 = 0 \]
\[ 2 - 7 - \frac{v_1}{2} - \frac{v_2}{4} = 0 \implies \frac{v_1}{2} + \frac{v_2}{4} = -5 \implies 2v_1 + v_2 = -20 \]

\[ v_1 - v_2 = -2 \]

\[ 2v_1 + v_2 = -20 \]
\[ v_1 - v_2 = -2 \]

\[ 3v_1 = -22 \implies v_1 = -\frac{22}{3} = -7.33V \]
\[ \implies v_2 = -\frac{22}{3} + 2 = -\frac{16}{3} = -5.33V \]
Nodal Analysis: Supernode

• Find the node voltages of the circuit below.
Nodal Analysis: Supernode

- Find the node voltages of the circuit below.
- Apply **KCL** to the two supernodes.

At **supernode 1-2**:

\[
\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}
\]

\[
5v_1 + v_2 - v_3 - 2v_4 = 60
\]
Nodal Analysis: Supernode

• Find the node voltages of the circuit below.
• Apply **KCL** to the two supernodes.

**At supernode 3-4:**

\[
\frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}
\]

\[
4v_1 + 2v_2 - 5v_3 - 16v_4 = 0
\]
**Nodal Analysis: Supernode**

- Find the node voltages of the circuit below.
- Apply **KVL** around the loops:

**Loop 1:** \[-v_1 + 20 + v_2 = 0\]
\[v_1 - v_2 = 20\]

**Loop 2:** \[-v_3 + 3v_x + v_4 = 0\]
\[v_x = v_1 - v_4\]
\[3v_1 - v_3 - 2v_4 = 0\]

**Loop 3:** \[v_x - 3v_x + 6i_3 - 20 = 0\]
\[6v_1 - v_3 - 2v_4 = 80\]
Nodal Analysis: Supernode

- Find the node voltages of the circuit below.

- In **matrix** form:

\[
\begin{bmatrix}
5v_1 + v_2 - v_3 - 2v_4 = 60 \\
4v_1 + 2v_2 - 5v_3 - 16v_4 = 0 \\
v_1 - v_2 = 20 \\
3v_1 - v_3 - 2v_4 = 0 \\
6v_1 - v_3 - 2v_4 = 80
\end{bmatrix}
\]

(KCL from supernode 1-2)

(KCL from supernode 3-4)

(KVL from loop1)

(KVL from loop2)

(KVL from loop3)

Only 4 equations are enough (ignore the 5th equation)

\[
\begin{bmatrix}
5 & 1 & -1 & -2 \\
4 & 2 & -5 & -16 \\
1 & -1 & 0 & 0 \\
3 & 0 & -1 & -2
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4
\end{bmatrix} =
\begin{bmatrix}
60 \\
0 \\
20 \\
0
\end{bmatrix}
\]
Nodal Analysis: Supernode

• Find the node voltages of the circuit below.

• You can use Matlab to solve large matrix equations:

\[
\begin{bmatrix}
5 & 1 & -1 & -2 \\
4 & 2 & -5 & -16 \\
1 & -1 & 0 & 0 \\
3 & 0 & -1 & -2
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4
\end{bmatrix}
= 
\begin{bmatrix}
60 \\
0 \\
20 \\
0
\end{bmatrix}
\]

We now use MATLAB to solve the matrix Equation. The Equation on the left can be written as

\[
AVA = B \Rightarrow V = B/A = A^{-1}B
\]

Matlab Code

```matlab
>> A=[5 1 -1 -2; 
4 2 -5 -16; 
1 -1 0 0; 
3 0 -1 -2];
>> B= [60 0 20 0]';
>> V=inv(A)*B
V =
26.6667
6.6667
173.3333
-46.6667
```

\(v_1 = 26.67\text{ V}\)
\(v_2 = 6.67\text{ V}\)
\(v_3 = 173.33\text{ V}\)
\(v_4 = -46.67\text{ V}\)
Mesh Analysis

1. Mesh analysis: another procedure for analyzing circuits, applicable to **planar** circuits.

2. A Mesh is a loop which does not contain any other loops within it.

3. Nodal analysis applies KCL to find voltages in a given circuit, while **Mesh Analysis** applies **KVL** to calculate unknown currents.
Mesh Analysis

- A circuit is **planar** if it can be drawn on a plane with no branches crossing one another. Otherwise it is nonplanar.
- The circuit in (a) is planar, because the same circuit that is redrawn (b) has no crossing branches.
Mesh Analysis

- A nonplanar circuit.
Mesh Analysis

• **Steps to Determine Mesh Currents:**

1. Assign *mesh currents* $i_1, i_2, .., i_n$ to the $n$ meshes.

2. Apply **KVL** to each of the $n$ meshes. Use **Ohm’s law** to express the voltages in terms of the mesh currents.

3. Solve the resulting $n$ *simultaneous equations* to get the mesh currents.
Mesh Analysis

- A circuit with two meshes.
Mesh Analysis

- A circuit with two meshes.

- Apply KVL to each mesh. For mesh 1,

\[-V_1 + R_1i_1 + R_3(i_1 - i_2) = 0\]

\[(R_1 + R_3)i_1 - R_3i_2 = V_1\]

- For mesh 2,

\[R_2i_2 + V_2 + R_3(i_2 - i_1) = 0\]

\[-R_3i_1 + (R_2 + R_3)i_2 = -V_2\]
Mesh Analysis

- A circuit with two meshes.

- Solve for the mesh currents.

\[
\begin{bmatrix}
R_1 + R_3 & -R_3 \\
-R_3 & R_2 + R_3
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
=
\begin{bmatrix}
V_1 \\
-V_2
\end{bmatrix}
\]

- Use \( i \) for a mesh current and \( I \) for a branch current.

It's evident from the circuit that:

\[
I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2
\]
Mesh Analysis: **Example 3.5**

- A circuit Find the branch currents $I_1$, $I_2$, and $I_3$ using mesh analysis.
Mesh Analysis: Example 3.5

• For mesh 1,

\[-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0\]

\[3i_1 - 2i_2 = 1\]

• For mesh 2,

\[6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0\]

\[i_1 = 2i_2 - 1\]

• We can find \(i_1\) and \(i_2\) by substitution method or Cramer’s rule. Then,

\[I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2\]
Mesh Analysis: Example 3.6

- Use mesh analysis to find the current $I_o$ in the circuit below.
Mesh Analysis: Example 3.6

• Apply KVL to each mesh. For mesh 1,

\[-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0\]

\[11i_1 - 5i_2 - 6i_3 = 12\]

• For mesh 2,

\[24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0\]

\[-5i_1 + 19i_2 - 2i_3 = 0\]
Mesh Analysis: Example 3.6

- For mesh 3,

\[ 4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0 \]

At node A, \( I_0 = I_1 - i_2 \),

\[ 4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0 \]

\[ -i_1 - i_2 + 2i_3 = 0 \]

- In matrix from:

\[
\begin{bmatrix}
11 & -5 & -6 \\
-5 & 19 & -2 \\
-1 & -1 & 2 \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
\end{bmatrix} = 
\begin{bmatrix}
12 \\
0 \\
0 \\
\end{bmatrix}
\]

- we can calculate \( i_1, i_2 \) and \( i_3 \) by Cramer’s rule, and find \( I_0 \).
Mesh Analysis: with Current Source

- Consider the following circuit with a current source.
Mesh Analysis: with Current Source

- Possible cases of having a current source.
  - **Case 1**
    - Current source exist only in one mesh
      - mesh 2: $i_2 = -5 \text{ A}$
      - mesh 1: $-10 + 4i_1 + 6(i_1 - i_2) = 0 \implies i_1 = -2 \text{ A}$
    - One mesh variable is reduced
  - **Case 2**
    - Current source exists between two meshes, a super-mesh is obtained.
Mesh Analysis: with Current Source

- Possible cases of having a current source.
  - A supermesh is considered when two meshes have a (dependent/independent) current source in common.
Mesh Analysis: with Current Source

- Applying KVL in the supermesh below:

\[ i_2 = i_1 + 6 \]

- Apply KCL at node 0 above:

\[ i_1 = -3.2 \text{ A}, \quad i_2 = 2.8 \text{ A} \]

\[ -20 + 6i_1 + 10i_2 + 4i_2 = 0 \]

\[ 6i_1 + 14i_2 = 20 \]
Mesh Analysis: with Current Source

- Properties of a Supermesh

1. The current is not completely ignored
   - provides the constraint equation necessary to solve for the mesh current.

2. A supermesh has no current of its own.

3. Several current sources in adjacency form a bigger supermesh.

4. A supermesh requires the application of both KVL and KCL.
Mesh Analysis: with Current Source

- Apply KVL in the supermesh (mesh 1 + mesh 2 + mesh 3):
  \[2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0\]
  \[i_1 + 3i_2 + 6i_3 - 4i_4 = 0\]

- Apply KCL at node P:
  \[i_2 = i_1 + 5\]

- Apply KCL at node Q:
  \[i_2 = i_3 + 3i_o\]
  \[i_o = -i_4\]
  \[i_2 = i_3 - 3i_4\]

- 4 equations for 4 variables. Using Cramer's Rule

\[
\begin{bmatrix}
1 & 3 & 6 & -4 \\
0 & 0 & -4 & 5 \\
-1 & 1 & 0 & 0 \\
0 & 1 & -1 & 3
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-5 \\
5 \\
0
\end{bmatrix}
\]

- Apply KVL in mesh 4:
  \[2i_4 + 8(i_4 - i_3) + 10 = 0\]
  \[5i_4 - 4i_3 = -5\]

\[i_1 = -7.5 \text{ A}, \quad i_2 = -2.5 \text{ A}, \quad i_3 = 3.93 \text{ A}, \quad i_4 = 2.143 \text{ A}\]
Nodal and Mesh Analysis by Inspection

- The analysis equations can be obtained by direct inspection

a) For circuits with only resistors and independent current sources. (nodal analysis)

b) For planar circuits with only resistors and independent voltage sources. (mesh analysis)
Nodal and Mesh Analysis by Inspection

a) For circuits with only resistors and independent current sources. (nodal analysis).

- Consider the following example:
- The circuit has two nonreference nodes and the node equations:

\[ I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2) \]
\[ I_2 + G_2 (v_1 - v_2) = G_3 v_2 \]

- In Matrix form:

\[
\begin{bmatrix}
G_1 + G_2 & -G_2 \\
-G_2 & G_2 + G_3
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
= 
\begin{bmatrix}
I_1 - I_2 \\
I_2
\end{bmatrix}
\]
Nodal and Mesh Analysis by Inspection

In general, if a circuit with independent current sources has $N$ nonreference nodes, the node-voltage equations can be written in terms of conductances as:

$$
\begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1N} \\
G_{21} & G_{22} & \cdots & G_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
G_{N1} & G_{N2} & \cdots & G_{NN}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_N
\end{bmatrix}
= 
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_N
\end{bmatrix}
\quad \Rightarrow \quad Gv = i
$$

- $G_{kk}$ is the sum of the conductances connected to node $k$
- $G_{kj} = G_{jk}$ is the negative of the sum of the conductances directly connecting nodes $k$ and $j$, $k \neq j$
- $v_k$ is the unknown voltage at node $k$
- $i_k$ is the sum of all independent current sources directly connected to node $k$, with currents entering the node treated as positive

$G$ is called the conductance matrix, $v$ is the output vector, and $i$ is the input vector.
Nodal and **Mesh Analysis by Inspection**

b) For planar circuits with only resistors and independent voltage sources. (mesh analysis)

- Consider the following example:

- The circuit has two meshes and the mesh equations:

- In Matrix form:

\[
\begin{bmatrix}
R_1 + R_3 & -R_3 \\
-R_3 & R_2 + R_3
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
=
\begin{bmatrix}
v_1 \\
-v_2
\end{bmatrix}
\]
Nodal and **Mesh Analysis by Inspection**

- In general, if a circuit has $N$ meshes, mesh-current equations can be expressed in terms of resistances as:

$$
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1N} \\
R_{21} & R_{22} & \cdots & R_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
R_{N1} & R_{N2} & \cdots & R_{NN}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_N
\end{bmatrix} =
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_N
\end{bmatrix} \quad \Rightarrow \quad Ri = v
$$

- $R_{kk}$ = Sum of the resistances in mesh $k$
- $R_{kj} = R_{jk} =$ Negative of the sum of the resistances in common with meshes $k$ and $j$, $k \neq j$
- $i_k =$ Unknown mesh current for mesh $k$ in the clockwise direction
- $v_k =$ Sum taken clockwise of all independent voltage sources in mesh $k$, with voltage rise treated as positive

- **$R$** is called the *resistance matrix*, **$i$** is the output vector, and **$v$** is the input vector.
Nodal and Mesh Analysis by Inspection

Example 3.8: Write the node-voltage matrix equations in the following circuit.
Nodal and Mesh Analysis by Inspection

Example 3.8: Write the node-voltage matrix equations in the following circuit.

The circuit has 4 nonreference nodes, so the diagonal terms of $G$ are:

$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \quad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

The off-diagonal terms of $G$ are:

$$G_{12} = -\frac{1}{5} = -0.2, \quad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \quad G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \quad G_{32} = -0.125, \quad G_{34} = -0.125$$

$$G_{41} = 0, \quad G_{42} = -1, \quad G_{43} = -0.125$$
Nodal and Mesh Analysis by Inspection

- **Example 3.8:** Write the node-voltage matrix equations in the following circuit.

- The input current vector \( \mathbf{i} \) in amperes:

\[
\begin{align*}
  i_1 &= 3, \\
  i_2 &= -1 - 2 = -3, \\
  i_3 &= 0, \\
  i_4 &= 2 + 4 = 6
\end{align*}
\]

- The node-voltage equations can be calculated by:

\[
\begin{bmatrix}
0.3 & -0.2 & 0 & 0 \\
-0.2 & 1.325 & -0.125 & -1 \\
0 & -0.125 & 0.5 & -0.125 \\
0 & -1 & -0.125 & 1.625
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
  v_4
\end{bmatrix}
= 
\begin{bmatrix}
  3 \\
  -3 \\
  0 \\
  6
\end{bmatrix}
\]
Example 3.9: Write the mesh-current equations in the following circuit.
Nodal and Mesh Analysis by Inspection

**Example 3.9:** Write the mesh-current equations in the following circuit.

- The input voltage vector $\mathbf{v}$ in volts:
  
  \[
  v_1 = 4, \quad v_2 = 10 - 4 = 6, \\
  v_3 = -12 + 6 = -6, \quad v_4 = 0, \quad v_5 = -6
  \]

- The mesh-current equations are:
  \[
  \begin{bmatrix}
  9 & -2 & -2 & 0 & 0 \\
  -2 & 10 & -4 & -1 & -1 \\
  -2 & -4 & 9 & 0 & 0 \\
  0 & -1 & 0 & 8 & -3 \\
  0 & -1 & 0 & -3 & 4 \\
  \end{bmatrix}
  \begin{bmatrix}
  i_1 \\
  i_2 \\
  i_3 \\
  i_4 \\
  i_5 \\
  \end{bmatrix}
  =
  \begin{bmatrix}
  4 \\
  6 \\
  -6 \\
  0 \\
  -6 \\
  \end{bmatrix}
  \]
Nodal versus Mesh Analysis

• Both nodal and mesh analyses provide a systematic way of analyzing a complex network.
• The choice of the better method dictated by two factors.

  ▪ **First factor**: nature of the particular network.

    ➢ The key is to select the method that results in the smaller number of equations.

  ▪ **Second factor**: information required.
Nodal versus Mesh Analysis: **Summary**

- Both **nodal** and **mesh analyses** provide a **systematic way** of analyzing a complex network.
- The choice of the better method is dictated by **two factors**.

1. **Nodal analysis**: Apply KCL at the nonreference nodes.  
   (The circuit with fewer node equations)
2. **A supernode**: Voltage source between two nonreference nodes.
3. **Mesh analysis**: Apply KVL for each mesh.  
   (The circuit with fewer mesh equations)
4. **A supermesh**: Current source between two meshes.
Application: DC Transistor Circuits: BJT Circuit Models

(a) An npn transistor,
(b) dc equivalent model.
Example 3.13: For the BJT circuit in Fig. 3.43, $\beta = 150$ and $V_{BE} = 0.7 \text{ V}$. Find $v_0$. 

![Diagram of BJT circuit with $\beta = 150$ and $V_{BE} = 0.7 \text{ V}$]
Example 3.13: For the BJT circuit in Fig. 3.43, $\beta=150$ and $V_{BE} = 0.7 \text{ V}$. Find $v_0$.

- Use mesh analysis or nodal analysis