DC Circuits:

Circuit Theorems

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Circuit Theorems

- Introduction
- Linearity Property
- Superposition
- Source Transformations
- Thevenin’s Theorem
- Norton’s Theorem
- Maximum Power Transfer
Introduction

A large complex circuits

Simplify circuit analysis

Circuit Theorems

- Thevenin’s theorem
- Circuit linearity
- Source transformation

- Norton theorem
- Superposition
- Max. power transfer
Linearity Property

- Homogeneity property (Scaling)
  \[ i \rightarrow v = iR \]
  \[ ki \rightarrow kv = kiR \]

- Additivitiy property
  \[ i_1 \rightarrow v_1 = i_1R \]
  \[ i_2 \rightarrow v_2 = i_2R \]
  \[ i_1 + i_2 \rightarrow (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2 \]
A **linear circuit** is one whose output is linearly related (directly proportional) to its input.

Linearity Property

![Diagram of a linear circuit](image)
A linear circuit consists of:

- **Linear elements** (i.e. \( R=5 \, \Omega \))
- **Linear dependent sources** (i.e. \( v_s=6I_o \, V \))
- **Independent sources** (i.e. \( v_s=12 \, V \))

\[
\begin{align*}
v_s = 10V & \rightarrow i = 2A \\
v_s = 1V & \rightarrow i = 0.2A \\
v_s = 5mV & \leftrightarrow i = 1mA
\end{align*}
\]

\[
p = i^2R = \frac{V^2}{R} : \text{nonlinear}
\]
Linearity Property

**Example 4.1:** For the circuit in below find $I_o$ when $v_s=12\,\text{V}$ and $v_s=24\,\text{V}$.
Linearity Property

Example 4.1: For the circuit in below find $I_o$ when $v_s=12\text{V}$ and $v_s=24\text{V}$.

KVL

\begin{align*}
12i_1 - 4i_2 + v_s &= 0 \quad \text{(1)} \\
-4i_1 + 16i_2 - 3\nu_x - v_s &= 0 \quad \text{(2)} \\
\nu_x &= 2i_1
\end{align*}

Eq(2) becomes

\begin{align*}
-10i_1 + 16i_2 - v_s &= 0 \quad \text{(3)}
\end{align*}

Using Eqs(1) and (3) we get

\begin{align*}
2i_1 + 12i_2 &= 0 \quad \implies \quad i_1 = -6i_2
\end{align*}
Linearity Property

Example 4.1: For the circuit in below find $I_o$ when $v_s=12\text{V}$ and $v_s=24\text{V}$.

From Eq(1) we get: $-76i_2 + v_s = 0 \implies i_2 = \frac{v_s}{76}$

when $v_s = 12\text{V}$

$I_o = i_2 = \frac{12}{76} \text{A}$

when $v_s = 24\text{V}$

$I_o = i_2 = \frac{24}{76} \text{A}$

Showing that when the source value is doubled, $I_o$ doubles.
Linearity Property

Example 4.2: Assume $I_o = 1\text{A}$ and use linearity to find the actual value of $I_o$ in the following circuit.
Linearity Property

Example 4.2: Assume $I_o = 1\text{A}$ and use linearity to find the actual value of $I_o$ in the following circuit.

If $I_0 = 1\text{A}$, then $v_1 = (3 + 5)I_0 = 8\text{V}$

$I_1 = v_1 / 4 = 2\text{A}$, \hspace{1cm} $I_2 = I_1 + I_0 = 3\text{A}$

$V_2 = V_1 + 2I_2 = 8 + 6 = 14\text{V}$, $I_3 = \frac{V_2}{7} = 2\text{A}$

$I_4 = I_3 + I_2 = 5\text{A}$ \Rightarrow \hspace{1cm} I_S = 5\text{A}$

$I_0 = 1\text{A} \rightarrow I_S = 5\text{A}$

$I_0 = 3\text{A} \leftarrow I_S = 15\text{A}$
Superposition

• The **superposition principle** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each **independent** source acting alone.

• **Turn off, kill, inactive source(s):**
  - independent voltage source: 0 V (short circuit)
  - independent current source: 0 A (open circuit)

• **Dependent sources are left intact.**
Superposition

• Steps to apply superposition principle:

1. **Turn off** all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. **Repeat** step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.
Superposition

- How to turn off independent sources:
  - Turn off voltages sources = short voltage sources; make it equal to zero voltage
  - Turn off current sources = open current sources; make it equal to zero current

- Superposition involves more work but simpler circuits.
- Superposition is not applicable to the effect on power.
Superposition

• **Example 4.3:** Use the superposition theorem to find \( v \) in the circuit below.
Superposition

- **Example 4.3**: Use the superposition theorem to find \( v \) in the circuit below.

Since there are two sources,

Let \( v = v_1 + v_2 \)

**Voltage division to get**

\[
v_1 = \frac{4}{4+8} (6) = 2V
\]

**Current division, to get**

\[
i_3 = \frac{8}{4+8} (3) = 2A
\]

Hence \( v_2 = 4i_3 = 8V \)

And we find \( v = v_1 + v_2 = 2 + 8 = 10V \)
Superposition

• Example 4.4: Find $I_o$ in the circuit below using superposition.
Superposition

- **Example 4.4:** Find $I_o$ in the circuit below using superposition.

  **Turn off** 20V voltage source:
Superposition

• **Example 4.4:** Find $I_o$ in the circuit below using superposition.

**Turn off** 4A current source:

![Circuit Diagram](image)

(b)
Source Transformation

- A source transformation is the process of replacing a voltage source $v_s$ in series with a resistor $R$ by a current source $i_s$ in parallel with a resistor $R$, or vice versa.

\[ v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R} \]
Source Transformation

• Equivalent Circuits:

\[ i = \frac{v - v_s}{R}, \quad v = -i_R - i_s \]

\[ i_s = \frac{v - v_s}{R} \]

\[ v = i R \]

\[ i = \frac{v - v_s}{R} \]

\[ v = -i_s - v_s \]
Source Transformation

- **Source transformation**: Important points

1. Arrow of the current source is directed toward the positive terminal of the voltage source.

2. Transformation is not possible when:
   - ideal voltage source \((R = 0)\)
   - ideal current source \((R = \infty)\)
Source Transformation

**Example 4.6:** Use source transformation to find $v_o$ in the circuit below.
Source Transformation

- **Example 4.6**: Use source transformation to find $v_o$ in the circuit below.

![Circuit Diagram](image)

Fig. 4.18
Source Transformation

**Example 4.6:** Use source transformation to find $v_o$ in the circuit below.

we use current division in Fig. 4.18(c) to get

\[
i = \frac{2}{2 + 8} (2) = 0.4 \text{A}
\]

and

\[
v_o = 8i = 8(0.4) = 3.2 \text{V}
\]
Source Transformation

**Example 4.7:** Find $v_x$ in the circuit below using source transformation.
Source Transformation

• **Example 4.7:** Find $v_x$ in the circuit below using source transformation.

![Circuit Diagram](image)

KVL around the loop in Fig (b)

$$-3 + 5i + v_x + 18 = 0 \quad (1)$$

Applying KVL to the loop containing only the 3V voltage source, the 1Ω resistor, and $v_x$ yields:

$$-3 + 1i + v_x = 0 \Rightarrow v_x = 3 - i \quad (2)$$
Source Transformation

• **Example 4.7:** Find $v_x$ in the circuit below using source transformation.

Substituting Eq.(2) into Eq.(1), we obtain

$$15 + 5i + 3 = 0 \Rightarrow i = -4.5\,\text{A}$$

Alternatively

$$-v_x + 4i + v_x + 18 = 0 \Rightarrow i = -4.5\,\text{A}$$

thus

$$v_x = 3 - i = 7.5\,\text{V}$$
Thevenin’s Theorem

- **Thevenin’s theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a **voltage source** \( V_{\text{Th}} \) in series with a **resistor** \( R_{\text{Th}} \) where \( V_{\text{Th}} \) is the open circuit voltage at the terminals and \( R_{\text{Th}} \) is the input or equivalent resistance at the terminals when the **independent sources are turned off**.
Thevenin’s Theorem

• Property of Linear Circuits

Any two-terminal Linear Circuits

\[ i \]
\[ + \]
\[ \nu \]
\[ - \]

Slope = \( \frac{1}{R_{Th}} \)

\[ V_{th} \]

\[ I_{sc} \]
Thevenin’s Theorem

- Replacing a **linear two-terminal circuit (a)** by its Thevenin equivalent circuit (b).

**Diagram:**

(a) Linear two-terminal circuit

(b) Thevenin equivalent circuit with voltage source $V_{Th}$ and resistance $R_{Th}$
Thevenin’s Theorem

• How to find Thevenin Voltage?
  ▪ Equivalent circuit: same voltage-current relation at the terminals.

\[ V_{Th} = v_{oc} : \text{ open circuit voltage at } a - b \]
Thevenin’s Theorem

- How to find Thevenin Resistance?

\[ R_{Th} = R_{in} \] : input – resistance of the dead circuit at \( a - b \).

- \( a - b \) open circuited

- Turn off all independent sources

Linear circuit with all independent sources set equal to zero

\[ R_{Th} = R_{in} \]

(b)
Thevenin’s Theorem

- How to find Thevenin Resistance?
  - There are **two cases** in finding Thevenin Resistance $R_{Th}$.

**CASE 1**

- If the network has **no dependent sources**:
  - Turn off all independent sources.
  - $R_{Th}$: can be obtained via simplification of either parallel or series connection seen from a-b
Thevenin’s Theorem

• How to find Thevenin Resistance?
  ▪ There are **two cases** in finding Thevenin Resistance $R_{Th}$.

**CASE 2**

• If the network has **dependent sources**:
  ● Turn off all independent sources.
  ● **Apply a voltage source** $v_o$ at **a-b**
    
    $$R_{Th} = \frac{v_o}{i_o}$$

  ● **Alternatively, apply a current source** $i_o$ at **a-b**
    
    $$R_{Th} = \frac{v_o}{i_o}$$
Thevenin’s Theorem

• How to find Thevenin Resistance?

  ▪ The Thevenin resistance, $R_{Th}$, may be negative, indicating that the circuit has ability providing (supplying) power.
Thevenin’s Theorem

- After the Thevenin Equivalent is obtained, the simplified circuit can be used to calculate $I_L$ and $V_L$ easily.

**Simplified circuit**

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

**Voltage divider**
Thevenin’s Theorem

Example 4.8: Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals $a-b$. Then find the current through $R_L = 6, 16, \text{ and } 36 \, \Omega$. 

![Circuit Diagram](image)
Thevenin’s Theorem

• Example 4.8: Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals $a-b$. Then find the current through $R_L = 6, 16, \text{ and } 36 \, \Omega$.

Find $R_{Th}$:

$R_{Th} : 32\text{V voltage source } \rightarrow \text{short}$

$2\text{A current source } \rightarrow \text{open}$

\[
R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega
\]
Thevenin’s Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals $a-b$. Then find the current through $R_L = 6, 16,$ and $36 \, \Omega$.

**Find $V_{Th}$:**

\[ V_{Th} : \]

(1) **Mesh analysis**

\[-32 + 4i_1 + 12(i_1 - i_2) = 0, \quad i_2 = -2A \]

\[ \therefore i_1 = 0.5A \]

\[ V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V \]

(2) **Alternatively, Nodal Analysis**

\[ (32 - V_{Th})/4 + 2 = V_{Th}/12 \]

\[ \therefore V_{Th} = 30V \]
Thevenin’s Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals $a-b$. Then find the current through $R_L = 6,16$, and $36 \, \Omega$.

**Find $V_{Th}$:** (3) Alternatively, source transform

\[
\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}
\]

\[
96 - 3V_{Th} + 24 = V_{Th} \Rightarrow V_{Th} = 30 \text{V}
\]

*Thevenin Equivalent circuit:*
Thevenin’s Theorem

Example 4.8: Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals a-b. Then find the current through $R_L = 6, 16,$ and $36 \ \Omega$.

Calculate $I_L$:

$$i_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

- $R_L = 6 \rightarrow I_L = 30/10 = 3A$
- $R_L = 16 \rightarrow I_L = 30/20 = 1.5A$
- $R_L = 36 \rightarrow I_L = 30/40 = 0.75A$
Thevenin’s Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals $a-b$. 

![Circuit Diagram]

\[ 2v_x \]

\[ 2 \Omega \]

\[ 2 \Omega \]

\[ 5 \text{ A} \]

\[ 4 \Omega \]

\[ v_x \]

\[ + \]

\[ - \]

\[ 6 \Omega \]

\[ a \]

\[ b \]
Thevenin’s Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals $a-b$.

- (independent + dependent sources case)

**Find $R_{Th}$:** Use Fig (a):

- Independent source $\rightarrow 0$
- Dependent source $\rightarrow$ intact

$$v_o = 1V, \quad R_{Th} = \frac{v_o}{i_o} = \frac{1}{4 \Omega}$$

\[\text{Fig (a)}\]
Thevenin’s Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals \(a-b\).

**Find \(R_{Th}\):** For **Loop 1**:

\[-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2\]

But \(v_x = -4i_2 = i_1 - i_2\)

\[\therefore i_1 = -3i_2\]
Thevenin’s Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals $a-b$.

**Find $R_{Th}:$**

For **Loops 2 and 3:**

\[
4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0
\]

\[
6(i_3 - i_2) + 2i_3 + 1 = 0
\]

Solving these equations gives

\[
i_3 = -\frac{1}{6}A.
\]

But $i_o = -i_3 = \frac{1}{6}A$

\[
\therefore R_{Th} = \frac{1V}{i_o} = 6\Omega
\]
Thevenin’s Theorem

Example 4.9: Find the Thevenin equivalent of the circuit below at terminals a-b.

Find $V_{Th}$: Use Fig (b): Mesh analysis

Loop 1: $i_1 = 5 \text{ A}$

Loop 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$12i_2 - 2i_3 = 20$

Loop 3: $-2v_x + 2(i_3 - i_2) = 0$

$-2(4(i_1 - i_2)) + 2(i_3 - i_2) = 0$

$6i_2 + 2i_3 = 40$

$\therefore i_2 = 60/18$

$V_{Th} = v_{oc} = 6i_2 = 20\text{ V}$

Thevenin Equivalent circuit: 20 V
Thevenin’s Theorem

**Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

**Find $V_{Th}$:**

$$V_{Th} = 0$$

**Find $R_{Th}$:** Use Fig (b):

(dependent source only case)

Apply a **current source** $i_o$ at $a-b$

$$R_{Th} = \frac{v_o}{i_o}$$

Nodal analysis:

$$i_o + i_x = 2i_x + v_o / 4$$
Thevenin’s Theorem

Example 4.10: Determine the Thevenin equivalent circuit in Fig (a).

Find $R_{Th}$: Use Fig (b):

But

$$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2}$$

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4}$$

or $v_o = -4i_o$

Thus $R_{Th} = \frac{v_o}{i_o} = -4\Omega$ (Supplying power)
Thevenin’s Theorem

- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

  **Input circuit:**

  **Thevenin Equivalent circuit:**

  - When $R_{Th}$ is negative, you must evaluate (check) if the two circuits are equivalent or not!
Thevenin’s Theorem

- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

**Input circuit:** (a)

**Thevenin Equivalent circuit:**

**Check** with $R_L$ (9Ω) and voltage source (10V)
Thevenin’s Theorem

• Example 4.10: Determine the Thevenin equivalent circuit in Fig (a).

Input circuit:

\[ 8i_x + 4i_1 + 2(i_1 - i_2) = 0 \quad , \quad i_x = i_2 - i_1 \]
\[ -2i_1 + 6i_2 = 0 \quad , \quad i_1 = 3i_2 \quad (1) \]
\[ 2(i_2 - i_1) + 9i_2 + 10 = 0 \]
\[ 5i_2 = -10 \quad \Rightarrow \quad i_2 = -2 \text{A} \]

Thevenin Equivalent circuit:

\[ -4i + 9i + 10 = 0 \]
\[ 5i = -10 \quad \Rightarrow \quad i = -2 \text{A} \]

Load current in both circuits are equal.
So the Thevenin Equivalent circuit is OK.
Norton’s Theorem

- **Norton’s theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a **current source** $I_N$ in parallel with a **resistor** $R_N$.

- $I_N$ is the short-circuit current through the terminals and

- $R_N$ is the input or equivalent resistance at the terminals when the independent source are turn off.
Norton’s Theorem

- Property of Linear Circuits

\[ v = \frac{1}{R_N} i \]

Slope = \( \frac{1}{R_N} \)
Norton’s Theorem

• How to find Norton Current
  - Thevenin and Norton resistances are equal:
    \[ R_N = R_{\text{Th}} \]
  - Short circuit current from \( a \) to \( b \) gives the Norton current:
    \[ I_N = i_{sc} = \frac{V_{\text{Th}}}{R_{\text{Th}}} \]
Norton’s Theorem

- Thevenin or Norton equivalent circuit
  - The open circuit voltage $v_{oc}$ across terminals $a$ and $b$
  - The short circuit current $i_{sc}$ at terminals $a$ and $b$
  - The equivalent or input resistance $R_{in}$ at terminals $a$ and $b$ when all independent source are turn off.

\[
V_{Th} = v_{oc}
\]
\[
I_N = i_{sc}
\]
\[
R_{Th} = \frac{V_{Th}}{I_N} = R_N
\]
Norton’s Theorem

- **Example 4.11**: Find the Norton equivalent circuit of the circuit in Fig 4.39.
Norton’s Theorem

**Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

**Find $R_N$:** Use Fig (a):

\[
R_N = 5 \parallel (8 + 4 + 8)\\
= 5 \parallel 20 = \frac{20 \times 5}{25} = 4 \Omega
\]
Norton’s Theorem

- **Example 4.11**: Find the Norton equivalent circuit of the circuit in Fig 4.39.

**Find** $I_N$:

Use Fig (b):

*(short circuit terminals $a$ and $b)*

**Mesh**: $i_1 = 2A$, $20i_2 - 4i_1 - 12 = 0$

$i_2 = 1A = i_{sc} = I_N$

(b) *(ignore 5Ω. Because it is short circuit)*
Norton’s Theorem

**Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

**Find $I_N$:** Use Fig (c): Alternative Method

$$I_N = \frac{V_{Th}}{R_{Th}}$$

$V_{Th}$: (open circuit voltage accross terminals $a$ and $b$)

**Mesh analysis:**

$$i_3 = 2 \text{A}, \quad 25i_4 - 4i_3 - 12 = 0$$

$$\therefore i_4 = 0.8 \text{A}$$

$$\therefore v_{oc} = V_{Th} = 5i_4 = 4 \text{V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{A}$$
Norton’s Theorem

- **Example 4.11**: Find the Norton equivalent circuit of the circuit in Fig 4.39.

**Input circuit:**

```
2 A
```

```
8 Ω
```

```
4 Ω
```

```
5 Ω
```

```
12 V
```

**Norton’s Equivalent circuit:**

```
1 A
```

```
4 Ω
```

```
8 Ω
```

```
a
```

```
b
```
Norton’s Theorem

- **Example 4.12:** Using Norton’s theorem, find $R_N$ and $I_N$ of the circuit in Fig 4.43 at terminals $a-b$. 

![Circuit Diagram]

\[ 2i_x \]
\[ 5 \Omega \]
\[ 4 \Omega \]
\[ 10 \text{ V} \]
Norton’s Theorem

- **Example 4.12:** Using Norton’s theorem, find \( R_N \) and \( I_N \) of the circuit in Fig 4.43 at terminals \( a-b \).

**Find \( R_N \):** Use Fig (a):

- 4Ω resistor shorted
- 5Ω || \( v_o \) || 2\( i_x \): Parallel

Hence,

\[
i_x = 0 \text{A}, \quad i_0 = \frac{v_o}{5\Omega} = \frac{1\text{V}}{5\Omega} = 0.2 \text{A}
\]

\[
\therefore \quad R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega
\]
Norton’s Theorem

- **Example 4.12:** Using Norton’s theorem, find $R_N$ and $I_N$ of the circuit in Fig 4.43 at terminals $a-b$.

**Find $I_N$:**

- $4\Omega \parallel 10\Omega \parallel 5\Omega \parallel 2i_x$ : Parallel

\[
\begin{align*}
    i_x &= \frac{10 - 0}{4} = 2.5\text{A}, \\
    i_{sc} &= i_x + 2i_x = \frac{10}{5} + 2(2.5) = 7\text{A}
\end{align*}
\]

\[
\therefore I_N = 7\text{A}
\]
Norton’s Theorem

- **Example 4.12**: Using Norton’s theorem, find \( R_N \) and \( I_N \) of the circuit in Fig 4.43 at terminals \( a-b \).

**Input circuit:**

**Norton’s Equivalent circuit:**
Maximum Power Transfer

- The Thevenin equivalent is useful in finding the **maximum power** a linear circuit can deliver to a load.

\[ p = i^2 R_L = \left( \frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L \]
Maximum Power Transfer

- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load \((R_L = R_{TH})\).

\[
p = i^2 R_L = \left( \frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L
\]
Maximum Power Transfer

- \( P_{\text{max}} \) can be obtained when:

\[
\frac{dp}{dR_L} = 0
\]

\[
\frac{dp}{dR_L} = V_{TH}^2 \left[ \frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{((R_{TH} + R_L)^2)^2} \right] = 0
\]

\[
= V_{TH}^2 \left[ \frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right] = V_{TH}^2 \left[ \frac{(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} \right] = 0
\]

\( \Rightarrow (R_{TH} + R_L - 2R_L) = 0 \)

\( \Rightarrow (R_{TH} - R_L) = 0 \)

\( R_L = R_{TH} \)

\[
P_{\text{max}} = \frac{V_{TH}^2}{4R_{TH}}
\]
Maximum Power Transfer

- **Example 4.13**: Find the value of $R_L$ for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.
Maximum Power Transfer

**Example 4.13:** Find the value of $R_L$ for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

**Find $R_{Th}$:**

\[
R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9 \Omega
\]
Maximum Power Transfer

**Example 4.13:** Find the value of $R_L$ for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

**Find $V_{TH}$:**

$$-12 + 18i_1 - 12i_2, \quad i_2 = -2A, \quad i_1 = -\frac{2}{3}A$$

**KVL around the outer loop:**

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{TH} = 0 \Rightarrow V_{TH} = 22V$$

![Circuit Diagram]

$$R_L = R_{TH} = 9\Omega \quad \Rightarrow \quad p_{\text{max}} = \frac{V_{TH}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44W$$