Q1) Consider the circuit shown. Assume that the current source applied to the circuit is a sinusoidal source where $i_s(t)=6\cos 4t$ A.

a) (50%) Calculate the currents $i_1(t)$ and $i_2(t)$ shown and the phase angle between $i_2(t)$ and $i_1(t)$ (You may use KCL).

b) (50%) Calculate the voltage $v_1(t)$ and the impedances $Z_{in}$ and $Z_1$ (in real imaginary form) which are indicated on the circuit.

Using KCL, $I_s = I_1 + I_2 \quad \ldots \quad (1)$

$$I_2 = \frac{v_1 - v_2}{Z_C} = \frac{4I_1 - 2I_1}{-52} = \frac{2I_1}{-52} = \frac{J I_1}{-52} \quad \ldots \quad (2)$$

Substitute (2) to (1)

$$6 = I_1 + Ji_1 \quad \Rightarrow \quad I_1 = \frac{6}{1 + J} = \frac{6 - 45}{12} = 3 - J3$$

$$I_1 = 3\sqrt{2} \angle -45^\circ$$

$$i_1(t) = 4.24 \cos (4t - 45^\circ)$$

From (1) $I_2 = I_s - I_1 = 6 - 3 + J3 = 3 + J3 = 4.24 \angle 145^\circ$

$$i_2(t) = 4.24 \cos (4t + 45^\circ)$$

$\angle I_2 - \angle I_1 = $ Phase Difference $= 45^\circ - (-45^\circ) = 90^\circ$

b) $V_1 = 4I_1 = 12 - J12 = 12\sqrt{2} \angle -45^\circ = 17 \angle -45^\circ$

$$v_1(t) = 17 \cos (4t - 45^\circ)$$

$$Z_{in} = \frac{v_1}{I_s} = \frac{12 - J12}{6} = 2 - J2$$

$$Z_1 = \frac{v_1}{I_2} = \frac{12 - J12}{3 + J3} = \frac{12 \sqrt{2} \angle -45^\circ}{3\sqrt{2} \angle 45^\circ} = 4 \angle 90^\circ - J4$$

The impedance of the dependent source is also $-52 \angle$